

The Spacetime Quaternion and its vector space

Q8 group:

$$i \quad -i \quad j \quad -j \quad k \quad -k \quad 1 \quad -1$$

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k, \quad jk = i, \quad ki = j, \quad ji = -k, \quad kj = -i, \quad ik = -j$$

$$q = t + xi + yj + zk, \text{ where } t, x, y, z \in \mathbb{R}$$

3 space dimensions over 3 imaginary axis and 1 time dimension  $t$  as real axis obtained by:

$$\|t\| = \frac{S}{V} = \frac{\sqrt{-(xi)^2 - (yj)^2 - (zk)^2}}{c^2 = 1}, \text{ using the signature (1,3)}$$

Now, we know the length of  $t$  and its coordinates  $x, y, z$  over their versors  $i, j, k$  and its direction by

$$\vec{0x}, \quad \vec{0y}, \quad \vec{0z}$$

$$\sqrt{t^2 - z^2} = \zeta, \quad \left(\frac{\zeta}{2}\right)^2 + z^2 = \sqrt{\frac{t^2 - z^2}{4} + z^2}$$

$$\vec{t} + \vec{z} = \vec{u}, \text{ id est, } 2\sqrt{\frac{t^2 - z^2}{4} + z^2} = u, \text{ its coordinates are } x, y, 2z$$

$$\vec{u} + \vec{x} = \vec{v}, \text{ id est, } 2\sqrt{\frac{u^2 - x^2}{4} + x^2} = v$$

$$\vec{v} + \vec{y} = \vec{\zeta}, \text{ id est, } 2\sqrt{\frac{v^2 - y^2}{4} + y^2} = \zeta$$

Then

$$\zeta = \sqrt{t^2 + 3x^2 + 3y^2 + 3z^2}$$

Example:

$$x = 2, \quad y = 1, \quad z = 4$$

$$t = 4.582575695$$

$$u = 8.3066238629$$

$$v = 9$$

$$\zeta = 2t = 9.1651513899$$

Space and Time are inextricably linked. A Space variation implies the consequent variation of the Time and vice versa.

Now, we know the length of the spacetime quaternion. The coordinates of its point  $\zeta$  are

$$2xi, 2yj, 2zk$$

Thence we also know the properties of its directional vector  $\vec{0\zeta}$

Thus

$$q = a + bi + cj + dk$$

$$\begin{aligned} \zeta &= t + xi + yj + zk = \sqrt{t^2 + 3x^2 + 3y^2 + 3z^2} = \\ &= \sqrt{4x^2 + 4y^2 + 4z^2} \end{aligned}$$

Thence the Space-time Quaternion can use 3 Variables instead of 4 to get its position vector defined by its vector space consisting of 7 vectors:

$$\vec{0x}, \vec{0y}, \vec{0z}, t, u, v, \zeta$$

$$\zeta = \left[ -4 \left( (xi)^2 + (yj)^2 + (zk)^2 \right) \right]^{\frac{1}{2}}$$

The spacetime quaternion

3 IM variables which however also include the Real variable.

Space and Time are indissoluble.

is thus made of

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