

APPENDIX

[1]

from Geometric series to Mercator series :

$$\sum_{k=0}^x n^k = 1 + n + n^2 + \dots + n^x = \frac{1 - n^{x+1}}{1 - n}$$

$$\frac{1}{1+n} = 1 - n + n^2 - n^3 + \dots \quad | \quad |n| < 1, \quad x \rightarrow +\infty$$

$$\int_0^x \frac{dn}{1+n} = \int_0^x \frac{(1+n)^{-1}}{1+n} dn = \int_0^x (1 - n + n^2 - n^3 + \dots) dn$$

$$\left[\ln|1+n| \right]_0^x = \int_0^x dn - \int_0^x n \, dn + \int_0^x n^2 \, dn - \int_0^x n^3 \, dn + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}$$