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$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{n}$$

$$= \left[1 > \frac{1}{2}\right] +$$

$$+ \left[\left(\frac{1}{2}\right) = \frac{1}{2}\right] + \qquad 2^{n-1} = 1$$

$$+ \left[\left(\frac{1}{3} + \frac{1}{4}\right) > \frac{1}{2}\right] + \qquad 2^{n-1} = 2$$

$$+ \left[\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > \frac{1}{2}\right] + \qquad 2^{n-1} = 4$$

$$+ \left[\left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) > \frac{1}{2}\right] + \qquad 2^{n-1} = 8$$

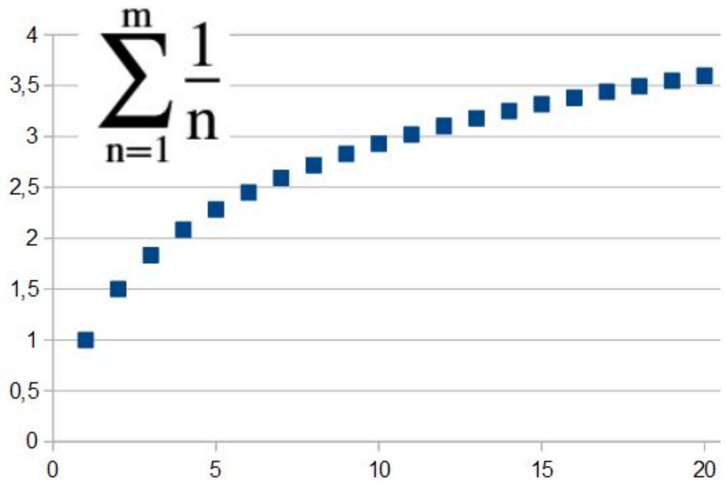
$$+ \left[\left(\ldots\ldots\ldots\ldots\frac{1}{32}\right) > \frac{1}{2}\right] + \qquad 2^{n-1} = 16$$

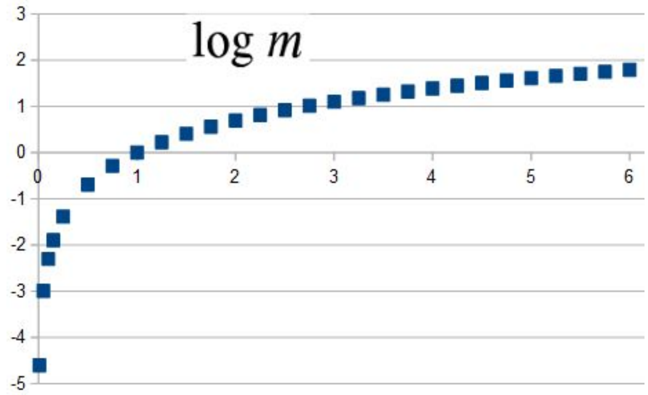
$$+ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ + \qquad 2^{n-1} = 32$$

$$\varepsilon \to 0 \ _{then} \ \mathrm{m} = +\infty - \varepsilon$$

$$\gamma = \sum_{n=1}^{m} \frac{1}{n} - \int_{1}^{m} \frac{1}{x}\, dx = \sum_{n=1}^{m} \frac{1}{n} - \left[\log x\right]_{1}^{m} = \sum_{n=1}^{m} \frac{1}{n} - \log m$$

partial harmonic series - Primitive of its function if $x \in \mathbb{N}$





$$\mathit{log}\ \mathrm{m} = \mathit{log}\ \mathrm{e}^{\mathrm{x}}$$

Transcendental number if X ***is Algebraic***

$$\mathrm{e}^{\mathrm{x}} = \left\lfloor \mathrm{e}^{\mathrm{x}} \right\rfloor + \mathrm{o}\left(\mathrm{e}^{\mathrm{x}}\right) \ \ \textit{where}\ \ \mathrm{o}\left(\mathrm{e}^{\mathrm{x}}\right) \in (0,1)$$

$$\gamma = \sum_{n=1}^{m} \frac{1}{n} - \log\left\{\left\lfloor \mathrm{e}^{\mathrm{x}} \right\rfloor + \mathrm{o}\left(\mathrm{e}^{\mathrm{x}}\right)\right\} \simeq 0.5772156649..................$$

$$\Gamma(\mathrm{n}) = \int_{0}^{\infty} \mathrm{x}^{\mathrm{n}-1} \mathrm{e}^{-\mathrm{x}} \ \ dx$$

$$\gamma = \mathrm{H}_{\mathrm{n}-1} - \psi(\mathrm{n})$$

to be continued ....

MEMORANDUM:

irrationality of e
if e = a/b then a(b-1)! should be an integer

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \sim 2.718282.....$$

$$e = \frac{a}{b} = \sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

$$\sum_{n=0}^{b} \frac{1}{n!} \geqslant 2 < 2.718282.....$$

$$a(b-1)! = b!\left[\sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}\right]$$

$$b!\sum_{n=0}^{b} \frac{1}{n!} < a(b-1)! < b!\sum_{n=0}^{b} \frac{1}{n!} + 1$$

$$\pi \sim 3.1415926536$$

$$n! = \prod_{i=1}^{n} i$$

$$(n+1)! = (n+1)n!$$

$$n \mid (n-1)! + 1$$

if $\frac{n(n+1)}{2}$ *is true, then if* $\frac{(n-1)n}{2}$ *is true, then* $\frac{(n+1)(n+2)}{2}$ *is true, hence all others*

$$\frac{n(n+1)}{2} = \frac{n(n-1)}{2} + n$$

$$(p-1)! \equiv -1 (\mathrm{mod}\; p)$$

$$n^{p-1} \equiv 1 (\mathrm{mod}\; p)$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \quad \sim 2.718282$$

$$\frac{n!}{k! \; (n-k)!}$$

$$(x+y)^n = \sum_{k=0}^{n} \left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle x^{n-k} y^k$$

$$\sum_{n=0}^{\infty} \frac{f^n x_0}{n!} (x - x_0)^n$$

$$\frac{fx - f\xi}{x - \xi} = f^{1}\Xi \quad \textit{where} \quad \Xi \in (\xi, x) \quad \textit{then}$$

$$fx = f\xi + f^{1}\Xi(x - \xi) \quad \textit{a case of Taylor + Lagrange remainder}$$

The first-order Taylor polynomial is the linear approximation of the function

$$\Psi x = D'\, ln\Gamma x = \frac{\Gamma' x}{\Gamma x}$$

$$\Gamma z = \int_{0}^{+\infty} t^{z-1} e^{-t}\ dt \qquad | Re(z) > 0$$

$$\Gamma(n+1) = n!$$

$$\beta(x,y) = \int_{0}^{1} t^{x-1} (1-t)^{y-1}\ dt \qquad | \ x,y \in c : Re(z) > 0$$

Stirling

$$\mathfrak{R}(z)^{+}, \quad \Gamma(z+1) = \prod(z) = z!$$

$$ln\Gamma(z) = \left(z - \frac{1}{2}\right)\ln z - z + \frac{\ln 2\pi}{2} + 2\int_{0}^{\infty} \frac{\arctan\frac{t}{z}}{e^{2\pi t} - 1}$$

Bernoulli numbers

$$fx = \frac{x}{e^{x} - 1} \qquad f'x = \frac{(1-x)e^{x} - 1}{\left(e^{x} - 1\right)^{2}} \qquad f''x = \frac{e^{x}\left((x-2)e^{x} + x + 2\right)}{\left(e^{x} - 1\right)^{3}}$$

$$B_n = \lim_{x \to 0} f^n x$$

$$B_0 = \lim_{x \to 0} \frac{x}{e^x - 1} = 1$$

$$B_1 = \lim_{x \to 0} \frac{(1-x)e^x - 1}{\left(e^x - 1\right)^2} = \frac{1}{2}$$

$$B_2 = \lim_{x \to 0} \frac{e^x\left((x-2)e^x + x + 2\right)}{\left(e^x - 1\right)^3} = \frac{1}{6}$$

$$\Gamma(x+1) =$$

$$= \int_0^\infty e^{-t} t^x \, dt = \left[-e^{-t} t^x\right]_0^\infty + \int_0^\infty e^{-t} x t^{x-1} \, dt = 0 + x \int_0^\infty e^{-t} t^{x-1} \, dt =$$

$$= x\Gamma(x) \qquad , x \in N \backslash \{0\}$$

$$\forall x \in N \backslash 0$$

$$x\Gamma(x) = x(x-1)\Gamma(x-1) = x(x-1)(x-2)\Gamma(x-2) = \ldots = x!$$

$$\begin{cases} 1! = 1 \\ (n+1)! = (n+1)n! \end{cases}$$

$$\begin{cases} \Gamma(1) = 1 \\ \Gamma(n+1) = n\Gamma(n) = n! \end{cases}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-t} t^{-\frac{1}{2}} \; dt$$

assuming $t = z^2 \qquad dt = 2z\,dz$

$$= 2\int_0^\infty e^{-z^2} z^{-1} z \; dz \qquad = \qquad \sqrt{\pi}$$

(Gaussian integral)

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

$$\Gamma\left(n + 1 + \frac{1}{2}\right) = \left(n + \frac{1}{2}\right)! = \frac{(2n+1)!!}{2^{n+1}}\sqrt{\pi}$$

$n \in N^+$

$$\left(-n + \frac{1}{2}\right)! = \Gamma\left(\frac{1}{2} - n + 1\right) = (-1)^{n-1} \frac{2^{n-1}}{(2n-3)!!}\sqrt{\pi}$$

$$\sum_{n=1}^{\infty}\frac{1}{n} - \int_1^\infty \frac{1}{n}\, dn = \sum_{n=1}^{\infty}\frac{1}{n} - \ln n_{\to\infty} = \int_1^\infty \frac{1}{\lfloor x \rfloor} - \frac{1}{x} \quad \sim 0.5772156649$$

$$\gamma = \sum_{k=1}^{n\to\infty} \frac{1}{k} - \ln n - o(1)$$ *where* $$ln\left(1+\frac{1}{n\to\infty}\right) \simeq o(1)$$

then, concerning the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \ln\left(\frac{n+1}{n}\right)\right)$$

assuming

$$x \in (0,1] \quad \to \quad \ln(1+x)$$

in the first-order Taylor polynomial:

$$\frac{x - ln(1+x)}{x-1-x} \quad = \quad ln(1+x) - x \quad = \quad f^1$$

we omit* dx *in the integrals

$$ln(1+x) = f^1 + x = x + \int ln(1+x) \quad - \int x =$$

$$= x - \frac{x^2}{2} + \int ln(1+x) = x - \frac{x^2}{2} + \left[x\, ln(1+x) - \int \frac{x+1-1}{1+x}\right] =$$

$$= x - \frac{x^2}{2} + \left[x ln(1+x) - \int 1 \ + \int \frac{1}{1+x}\right] =$$

$$= x - \frac{x^2}{2} + x\, ln(1+x) - x \ + ln(1+x) = \frac{x^2}{2x} = \frac{x}{2}$$

$$0 < x - ln(1+x) < x$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\ln\left(\frac{n+1}{n}\right)\right)$$ ***this series converges by the comparison criterion***

$$\gamma \;=\; \sum_{n=1}^{\substack{n\to\infty\\ n-1}}\frac{1}{n} \quad -\Psi(n)$$

$$\Psi(1) \;=\; \frac{e^{-t}}{-e^{-t}}$$ ***then*** $$-\gamma(1) \;=\; \gamma\Psi(1)$$

$$x^2\frac{d^2y}{dx^2} \;+ x\frac{dy}{dx} \;+\; y\left(x^2-z^2\right)$$ ***Bessel f***

$$\infty\leftarrow\bar{9}9999 \quad + \;\infty\leftarrow\bar{0}0001 \quad =0 \;\Longrightarrow \infty\leftarrow\bar{9}9999=-1$$

to be continued ....