

Disquisitions on exp(x).

$n \rightarrow \pm\infty$

$$e^e = e^{\left(1 + \frac{1}{n}\right)^n} = \sum_{m=0}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^{n^m}}{m!} = \sum_{j=0}^m \binom{m}{j} \frac{\left(1 + \frac{1}{n}\right)^{n^j}}{m^j} =$$

$$= \sum_{j=0}^m \frac{\prod_{h=0}^{j-1} m - h}{j!} \cdot \frac{\left(1 + \frac{1}{n}\right)^{nj}}{m^j} >$$

$$> \binom{\left(\frac{1}{0!}\right)^{\left(\frac{1}{0!}\right)}}{\mathbf{0!}} + \binom{\left(\frac{1}{1!}\right)^{\left(\frac{1}{1!}\right)}}{\mathbf{1!}} + \binom{\left(\frac{1}{3!}\right)^{\left(\frac{1}{3!}\right)}}{\mathbf{3!}} + \cdots + \binom{\left(\frac{1}{n!}\right)^{\left(\frac{1}{n!}\right)}}{\mathbf{n!}}$$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = \sum_{k=0}^{n \rightarrow \infty} \binom{n}{k} \left(\frac{x}{n}\right)^k, \quad x \in \mathbb{R}, \mathbb{C}$$

$$e^{iy}, \text{ then } iy = \log \sum_{n=0}^{\infty} \frac{(iy)^n}{n!}, \text{ where } y \in \mathbb{R}$$

$$\frac{1}{y} \log \sum_{n=0}^{\infty} \frac{(iy)^n}{n!} = \sqrt{-1}$$

$(iy)^n$ is an alternating series absolutely convergent

$$n! > (iy)^n, \text{ then } \frac{(iy)^n}{n!} < 1$$

$$\sum_{n=0}^{\infty} \frac{(iy)^n}{n!} \simeq 0$$

$$\ln(\sim 0^+) = -\lambda, \text{ where } \lambda \in \mathbb{R}^+ \text{ is very large}$$

$$-\frac{\lambda}{y} = i$$

$$-\lambda = \lambda \in \mathbb{R}^-$$

$$\sqrt{\frac{\lambda}{y}} = -1 \rightarrow y \in \mathbb{R}^+$$

$$e^{iy} = \cos y - \frac{\lambda}{y} \sin y = e^{\frac{\lambda}{y}y} = e^{\lambda}, \text{ BUT } \lambda \neq iy$$

$$q = x + yi + \xi j + \upsilon k$$

where

$$x, y, \xi, \upsilon \in \mathbb{R}$$

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k, \quad jk = i, \quad ki = j, \quad ji = -k, \quad kj = -i, \quad ik = -j$$

$$\begin{aligned} e^q &= e^x e^{yi} e^{\xi j} e^{\upsilon k} = \sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(yi)^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\xi j)^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\upsilon k)^n}{n!} \\ &= \left(1 + x + \frac{x^2}{2} + \dots\right) \left(1 + iy - \frac{y^2}{2} - \frac{iy^3}{6} + \frac{y^4}{24} + \frac{iy^5}{120} - \dots\right) (\dots)(\dots) \end{aligned}$$

trigonometric function

$$e^q = \sum_{n=0}^{\infty} \frac{\widehat{x}^n}{n!} \begin{bmatrix} c(y)c(\xi)c(\upsilon) & + jc(y)s(\xi)c(\upsilon) + \\ + is(y)c(\xi)c(\upsilon) & + ks(y)s(\xi)c(\upsilon) + \\ + kc(y)c(\xi)s(\upsilon) & + ic(y)s(\xi)s(\upsilon) - \\ -js(y)c(\xi)s(\upsilon) & -s(y)s(\xi)s(\upsilon) \end{bmatrix}$$

$$x + yi = a \quad , \quad x + \xi j = b \quad , \quad x + \upsilon k = c$$

$$\widehat{ab} = \theta \quad , \quad |a||b|\cos\theta = z \quad , \quad \widehat{zc} = \alpha \quad , \quad |z||c|\cos\alpha = \mathfrak{z}$$

$$\bar{q} = q^* = -\frac{q + i|q| + j|q| + k|q|}{2} \quad \text{then} \quad q^{-1} = \frac{q^*}{|q|^2}$$

to be continued...

$$\left(\{e^x\} = e^x - \lfloor e^x \rfloor\right) \in [0,1)$$

$$\begin{aligned} \ln n = \ln e^x &= \ln \left[\lfloor e^x \rfloor + \{e^x\} \right] = \ln \left[e^x \left(\frac{\lfloor e^x \rfloor + \{e^x\}}{e^x} \right) \right] \\ &= \ln \lfloor e^x \rfloor + \ln \left(1 + \frac{\{e^x\}}{\lfloor e^x \rfloor} \right) = \ln \lfloor e^x \rfloor + \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\{e^x\}}{\lfloor e^x \rfloor} \right)^n \end{aligned}$$

where

$$\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\{e^x\}}{\lfloor e^x \rfloor} \right)^n = \log(1 + \xi)$$

where normally $\xi = 1$ generates a particular Mercator series like an alternating harmonic CONVERGING series! (although not for absolute values).

but assuming

$$\int_1^2 \frac{1}{n} = \left[\ln n \right]_1^2 = \ln 2 = \ln(1 + e^x)$$

$$\xi = \frac{\{e^x\}}{\lfloor e^x \rfloor} = \frac{0}{1} \quad \xi = 1 = \frac{\{e^x\}}{\lfloor e^x \rfloor} = 0 \neq 1$$

sets the Mercator series to zero, but

by reductio ad absurdum, the Mercator series is not defined by $\log(n \in \mathbb{N})$ for $e^x \in \mathbb{N} \rightarrow \{\} = 0$ (mantissa = zero)

proof of the irrationality of e
if $e = a/b$ then $a(b-1)!$ should be an integer

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \sim 2.718281828459.....$$

$$e = \frac{a}{b} = \sum_{n=0}^b \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

$$\sum_{n=0}^b \frac{1}{n!} \gtrsim 2 < 2.718282.....$$

$$a(b-1)! = b! \left[\sum_{n=0}^b \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!} \right]$$

$$b! \sum_{n=0}^b \frac{1}{n!} < a(b-1)! < b! \sum_{n=0}^b \frac{1}{n!} + 1$$

from Geometric series to Mercator series :

$$\sum_{k=0}^x n^k = 1 + n + n^2 + \dots + n^x = \frac{1 - n^{x+1}}{1 - n}$$

$$\frac{1}{1+n} = 1 - n + n^2 - n^3 + \dots \quad | \quad |n| < 1, \quad x \rightarrow +\infty$$

$$\int_0^x \frac{dn}{1+n} = \int_0^x \frac{(1+n)^{-1}}{1+n} dn = \int_0^x (1 - n + n^2 - n^3 + \dots) dn$$

$$\left[\ln|1+n| \right]_0^x = \int_0^x dn - \int_0^x n dn + \int_0^x n^2 dn - \int_0^x n^3 dn + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}$$

a proof of the divergence of the harmonic series

$$\begin{aligned}
& \left[1 > \frac{1}{2} \right] + \\
+ & \left[\left(\frac{1}{2} \right) = \frac{1}{2} \right] + & 2^{n-1} = 1 \\
+ & \left[\left(\frac{1}{3} + \frac{1}{4} \right) > \frac{1}{2} \right] + & 2^{n-1} = 2 \\
+ & \left[\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \right) > \frac{1}{2} \right] + & 2^{n-1} = 4 \\
+ & \left[\left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} \right) > \frac{1}{2} \right] + & 2^{n-1} = 8 \\
+ & \left[\left(\dots \frac{1}{32} \right) > \frac{1}{2} \right] + & 2^{n-1} = 16 \\
+ & \dots + & 2^{n-1} = 32
\end{aligned}$$