14. Utility-Scale Experiment III - Part 1

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Approximate QPU time to run this experiment is 12 m 30 s.

In [1]:
```
import qiskit
qiskit.__version__
```

Out[1]: '1.3.0'

In [2]:
```
import qiskit_ibm_runtime
qiskit_ibm_runtime.__version__
```

Out[2]: '0.34.0'

In [3]:
```
import numpy as np
import rustworkx as rx
import matplotlib.pyplot as plt

from qiskit import QuantumCircuit
from qiskit.visualization import plot_histogram
from qiskit.visualization import plot_gate_map
from qiskit.transpiler.preset_passmanagers import generate_preset_pass_manager
from qiskit.providers import BackendV2
from qiskit.quantum_info import SparsePauliOp
from qiskit.transpiler import TranspileLayout

from qiskit_ibm_runtime import QiskitRuntimeService
from qiskit_ibm_runtime import Sampler, Estimator, Batch, SamplerOptions
```

Let us briefly review GHZ states, and what sort of distribution you might expect from `Sampler` applied to one. Then we will spell out the goal of this lesson explicitly.

GHZ state

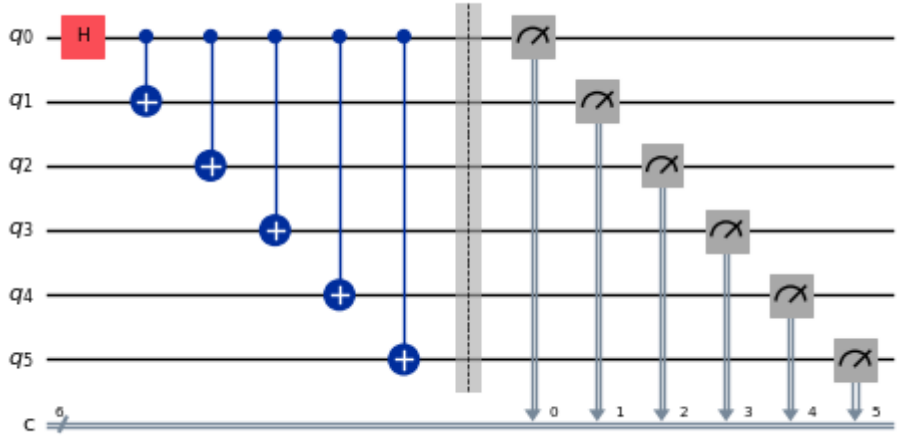
The GHZ state (Greenberger-Horne-Zeilinger state) for n qubits is defined as

$$\frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1\rangle^{\otimes n})$$

Naturally, it can be created for 6 qubits with the following quantum circuit.

In [4]:
```
N = 6
qc = QuantumCircuit(N, N)

qc.h(0)
for i in range(N-1):
    qc.cx(0,i+1)

#qc.measure_all()
qc.barrier()
qc.measure(list(range(N)), list(range(N)))

qc.draw(output="mpl", idle_wires=False, scale=0.5)
```

Out[4]:



In [5]:
```
print('Depth:', qc.depth())
```

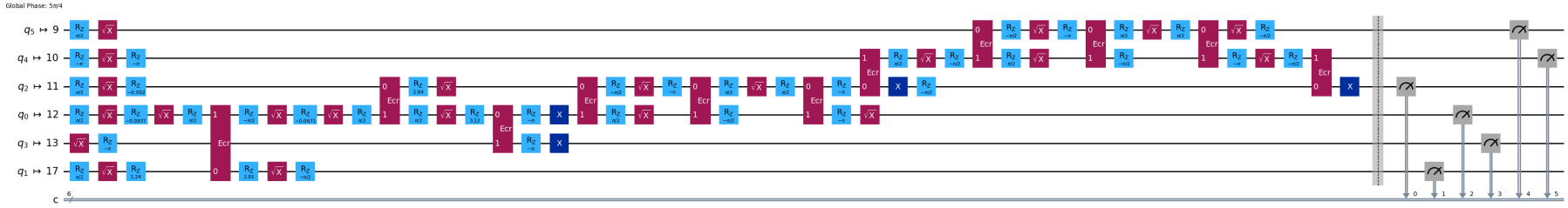
```
Depth: 7
```

The depth is not too large, though you know from previous lessons that you can do better. Let's pick a backend and transpile this circuit.

In [ ]:
```
service = QiskitRuntimeService()
backend = service.backend("ibm_kyiv")
# or
#backend = service.least_busy(operational=True)
#backend.name
```

In [6]:
```
pm = generate_preset_pass_manager(3, backend=backend)
qc_transpiled = pm.run(qc)
qc_transpiled.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[6]:



In [7]:
```
print('Depth:', qc_transpiled.depth())
print('Two-qubit Depth:', qc_transpiled.depth(filter_function=lambda x: x.operation.num_qubits==2))
```

```
Depth: 47
Two-qubit Depth: 11
```

Again the transpiled two-qubit depth is not too large. But to work with a GHZ state on more qubits, you will clearly need to think about optimizing the circuit. Let's run this using `Sampler` and see what a real quantum computer returns.

In [8]:
```
sampler = Sampler(mode=backend)
shots = 40000
job = sampler.run([qc_transpiled], shots = shots)
print("job id:", job.job_id())
```

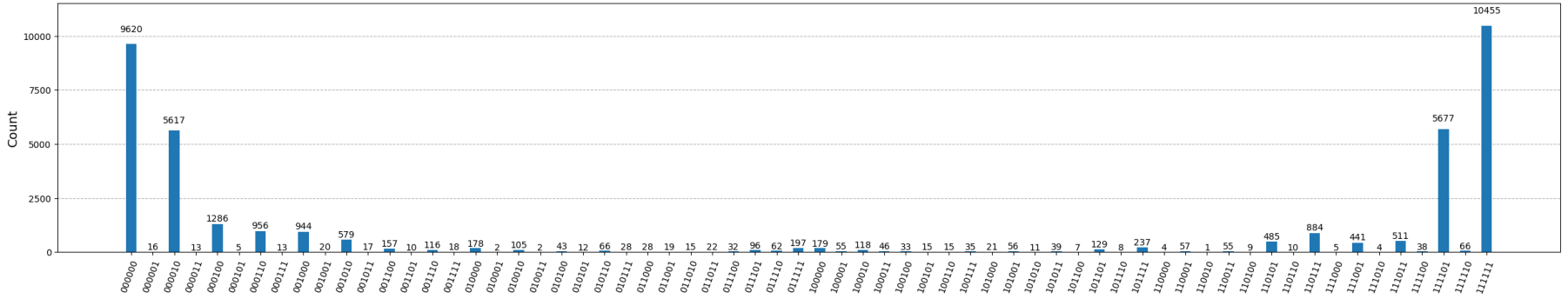
```
job id: cv3adfdhdzz0008mf3dg
```

In [8]:
```
job = service.job('cv3adfdhdzz0008mf3dg') # Use your job id shown above.
job.status()
```

Out[8]: 'DONE'

In [9]:
```
result = job.result()
plot_histogram(result[0].data.c.get_counts(), figsize=(30, 5))
```

Out[9]:



This is the result of the 6-qubit GHZ circuit. As you can see, the states of all $|0\rangle$'s and all $|1\rangle$'s do dominate, but the errors are substantial. Let's try to see how large a GHZ circuit you can make with an Eagle device, while still getting results where the correct states are at least more than 50% likely.

Your goal

Build a GHZ circuit for 20 qubits or more so that, upon measurement, **the fidelity of your GHZ state > 0.5.** Note:

- You need to use an Eagle device (`ibm_kyiv` , etc.) and set the shots number as 40,000.
- You should execute the GHZ circuit using the `execute_ghz_fidelity` function, and calculate the fidelity using the `check_ghz_fidelity_from_jobs` function.

This is intended as an independent exercise, in which you leverage what you have learned so far in this course.

In [6]:

```python
def execute_ghz_fidelity(
    ghz_circuit: QuantumCircuit,  # Quantum circuit to create GHZ state (Circuit after Routing or withou
    physical_qubits: list[int],  # Physical qubits to represent GHZ state
    backend: BackendV2,
    sampler_options: dict | SamplerOptions | None = None,
):
    N_SHOTS = 40_000
    N = len(physical_qubits)
    base_circuit = ghz_circuit.remove_final_measurements(inplace=False)
    # M_k measurement circuits
    mk_circuits = []
    for k in range(1, N + 1):
        circuit = base_circuit.copy()
        # change mesurement basis
        for q in physical_qubits:
            circuit.rz(-k * np.pi / N, q)
            circuit.h(q)
        mk_circuits.append(circuit)

    obs = SparsePauliOp.from_sparse_list(
        [("Z" * N, physical_qubits, 1)], num_qubits=backend.num_qubits
    )
    job_ids = []
    pm1 = generate_preset_pass_manager(1, backend=backend)
    org_transpiled = pm1.run(ghz_circuit)
    mk_transpiled = pm1.run(mk_circuits)
    with Batch(backend=backend):
        sampler = Sampler(options=sampler_options)
        sampler.options.twirling.enable_measure = True
        job = sampler.run([org_transpiled], shots=N_SHOTS)
        job_ids.append(job.job_id())
        print(f"Sampler job id: {job.job_id()}, shots={N_SHOTS}")
        estimator = Estimator()  # TREX is applied as default
        estimator.options.dynamical_decoupling.enable = True
        estimator.options.execution.rep_delay = 0.0005
        estimator.options.twirling.enable_measure = True
        job2 = estimator.run([(circ, obs) for circ in mk_transpiled], precision=1 / 100)
        job_ids.append(job2.job_id())
        print("Estimator job id:", job2.job_id())
```

In [7]:

```python
def check_ghz_fidelity_from_jobs(
    sampler_job,
    estimator_job,
    num_qubits,
    shots = 40_000,
):
    N = num_qubits
    sampler_result = sampler_job.result()
    counts = sampler_result[0].data.c.get_counts()
    all_zero = counts.get("0" * N, 0) / shots
    all_one = counts.get("1" * N, 0) / shots
    top3 = sorted(counts, key=counts.get, reverse=True)[:3]
    print(f"N={N}: |00..0>: {counts.get('0'*N, 0)}, |11..1>: {counts.get('1'*N, 0)}, |3rd>: {counts.get
    print(f"P(|00..0>)={all_zero}, P(|11..1>)={all_one}")

    estimator_result = estimator_job.result()
    non_diagonal = (1 / N) * sum(
        (-1) ** k * estimator_result[k - 1].data.evs for k in range(1, N + 1)
    )
    print(f"REM: Coherence (non-diagonal): {non_diagonal:.6f}")
    fidelity = 0.5 * (all_zero + all_one + non_diagonal)
    sigma = 0.5 * np.sqrt(
        (1 - all_zero - all_one) * (all_zero + all_one) / shots
        + sum(estimator_result[k].data.stds ** 2 for k in range(N)) / (N * N)
    )
    print(f"GHZ fidelity = {fidelity:.6f} ± {sigma:.6f}")
    if fidelity - 2 * sigma > 0.5:
        print("GME (genuinely multipartite entangled) test: Passed")
    else:
        print("GME (genuinely multipartite entangled) test: Failed")
    return {
        "fidelity": fidelity,
        "sigma": sigma,
        "shots": shots,
        "job_ids": [sampler_job.job_id(), estimator_job.job_id()],
    }
```

In this notebook, we will apply three strategies to creating good GHZ states using 16 qubits and 30 qubits. These approaches build on strategies you already know from previous lessons.

Strategy 1. Noise-aware qubit selection

We first specify a backend. Because we will be working extensively with the properties of a specific backend, it is a good idea to specify one backend, as opposed to using the `least_busy` option.

In [ ]:
```
backend = service.backend("ibm_kyiv")
twoq_gate = "ecr"
print(f"Device {backend.name} Loaded with {backend.num_qubits} qubits")
print(f"Two Qubit Gate: {twoq_gate}")
```

We are going to build a circuit involving many two-qubit gates. It makes sense for us to use the qubits that have the lowest errors when implementing those two-qubit gates. Finding the best "qubit chain" based on the reported 2q-gate errors is a not-trivial problem. But we can define a few functions to help us determine the best qubits to use.

In [9]:
```
coupling_map = backend.target.build_coupling_map(twoq_gate)
G = coupling_map.graph
```

In [10]:

```python
def to_edges(path): #create edges list from node paths
    edges = []
    prev_node = None
    for node in path:
        if prev_node is not None:
            if G.has_edge(prev_node, node):
                edges.append((prev_node, node))
            else:
                edges.append((node, prev_node))
        prev_node = node
    return edges


def path_fidelity(path, correct_by_duration: bool = True, readout_scale: float = None):
    """Compute an estimate of the total fidelity of 2-qubit gates on a path.
    If `correct_by_duration` is true, each gate fidelity is worsen by
    scale = max_duration / duration, i.e. gate_fidelity^scale.
    If `readout_scale` > 0 is supplied, readout_fidelity^readout_scale
    for each qubit on the path is multiplied to the total fielity.
    The path is given in node indices form, e.g. [0, 1, 2].
    An external function `to_edges` is used to obtain edge list, e.g. [(0, 1), (1, 2)]."""
    path_edges = to_edges(path)
    max_duration = max(backend.target[twoq_gate][qs].duration for qs in path_edges)

    def gate_fidelity(qpair):
        duration = backend.target[twoq_gate][qpair].duration
        scale = max_duration / duration if correct_by_duration else 1.0
        # 1.25 = (d+1)/d with d = 4
        return max(0.25, 1 - (1.25 * backend.target[twoq_gate][qpair].error)) ** scale

    def readout_fidelity(qubit):
        return max(0.25, 1 - backend.target["measure"][(qubit,)].error)

    total_fidelity = np.prod([gate_fidelity(qs) for qs in path_edges]) #two qubits gate fidelity for ea
    if readout_scale:
        total_fidelity *= np.prod([readout_fidelity(q) for q in path]) ** readout_scale # multipy readou
    return total_fidelity


def flatten(paths, cutoff=None):  # cutoff is for not making run time too large
```

```
    return [
        path
        for s, s_paths in paths.items()
        for t, st_paths in s_paths.items()
        for path in st_paths[:cutoff]
        if s < t
    ]
```

In [11]:
```
N = 16 # Number of qubits to use in the GHZ circuit
num_qubits_in_chain = N
```

We will use the functions above to find all the simple paths of N qubits between all pairs of nodes in the graph (Reference: all_pairs_all_simple_paths (https://www.rustworkx.org/apiref/rustworkx.all_pairs_all_simple_paths.html#rustworkx-all-pairs-all-simple-paths)).

Then, using the `path_fidelity` function created above, we will find the best qubit chain which has the largest path fidelity.

In [12]:
```
%%time
paths = rx.all_pairs_all_simple_paths(
    G.to_undirected(multigraph=False),
    min_depth=num_qubits_in_chain,
    cutoff=num_qubits_in_chain,
)
paths = flatten(paths, cutoff=25)  # If you have time, you could set a larger cutoff.
if not paths:
    raise Exception(
        f"No qubit chain with length={num_qubits_in_chain} exists in {backend.name}. Try smaller num_qu
    )

print(f"Selecting the best from {len(paths)} candidate paths")
from functools import partial
best_qubit_chain = max(paths, key=partial(path_fidelity, correct_by_duration=True, readout_scale=1.0))
assert len(best_qubit_chain) == num_qubits_in_chain
print(f"Predicted (best possible) process fidelity: {path_fidelity(best_qubit_chain)}")
```

```
Selecting the best from 6046 candidate paths
Predicted (best possible) process fidelity: 0.8916365774711382
CPU times: user 345 ms, sys: 5.59 ms, total: 351 ms
Wall time: 355 ms
```

In [13]:
```
1 np.array(best_qubit_chain)
```

Out[13]:
```
array([59, 60, 53, 41, 42, 43, 44, 45, 54, 64, 65, 66, 73, 85, 84, 83],
      dtype=uint64)
```

Let's plot the best qubit chain, shown in pink, in the coupling map diagram.

In [ ]:
```
1 qubit_color = []
2 for i in range(133):
3     if i in best_qubit_chain:
4         qubit_color.append("#ff00dd") #pink
5     else:
6         qubit_color.append("#8c00ff") #purple
7 plot_gate_map(backend, qubit_color=qubit_color, qubit_size=50, font_size=25, figsize=(6, 6))
```

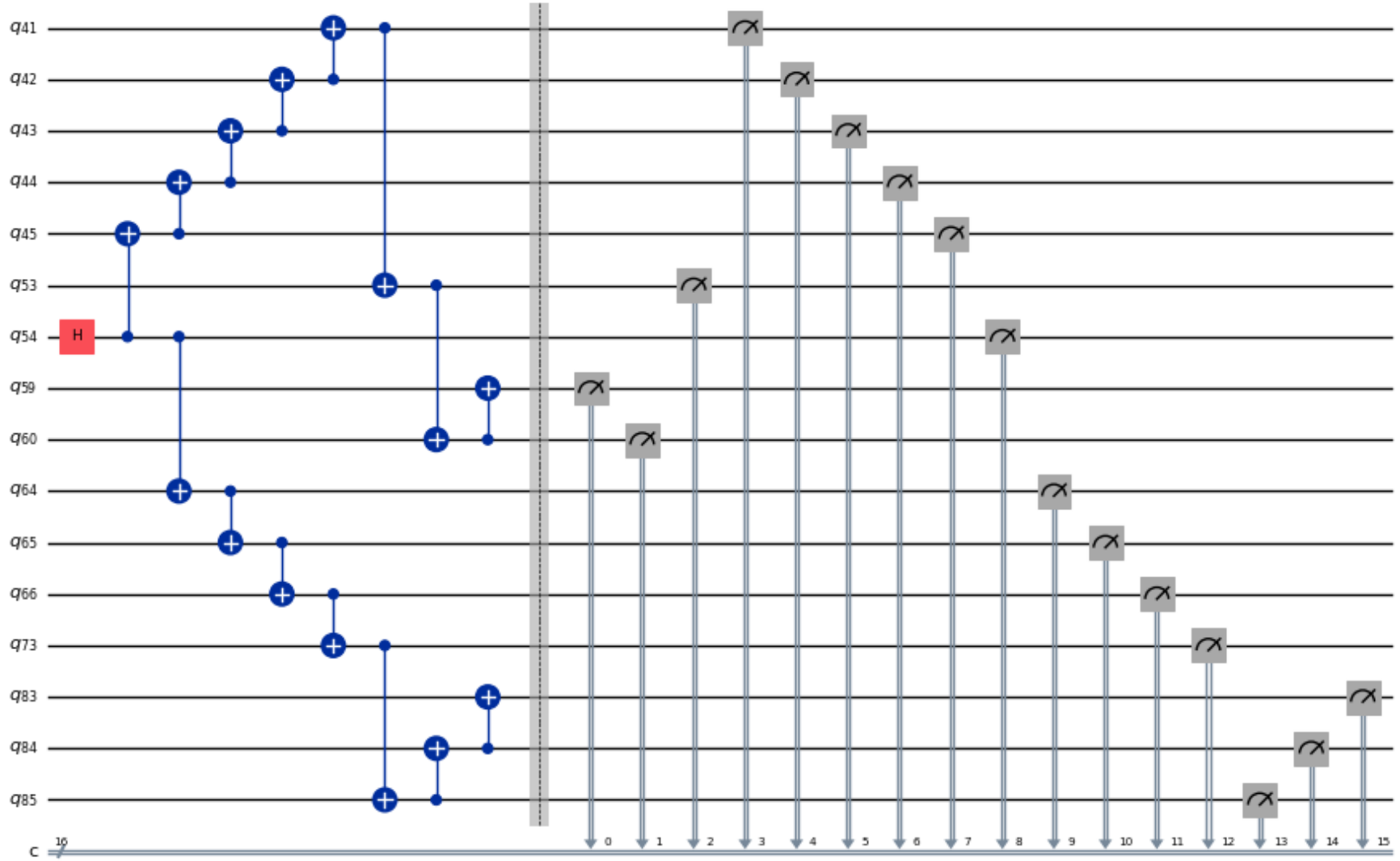
Build a GHZ circuit on the best qubit chain

We choose a qubit in the middle of the chain to first apply the H gate to. This should reduce the depth of the circuit by about half.

In [15]:

```
ghz1 = QuantumCircuit(max(best_qubit_chain)+1, N)
ghz1.h(best_qubit_chain[N//2])
for i in range(N//2, 0, -1):
    ghz1.cx(best_qubit_chain[i], best_qubit_chain[i-1])
for i in range(N//2, N-1, +1):
    ghz1.cx(best_qubit_chain[i], best_qubit_chain[i+1])
ghz1.barrier()  # for visualization
ghz1.measure(best_qubit_chain, list(range(N)))
ghz1.draw(output="mpl", idle_wires=False, scale=0.5, fold=-1)
```

Out[15]:

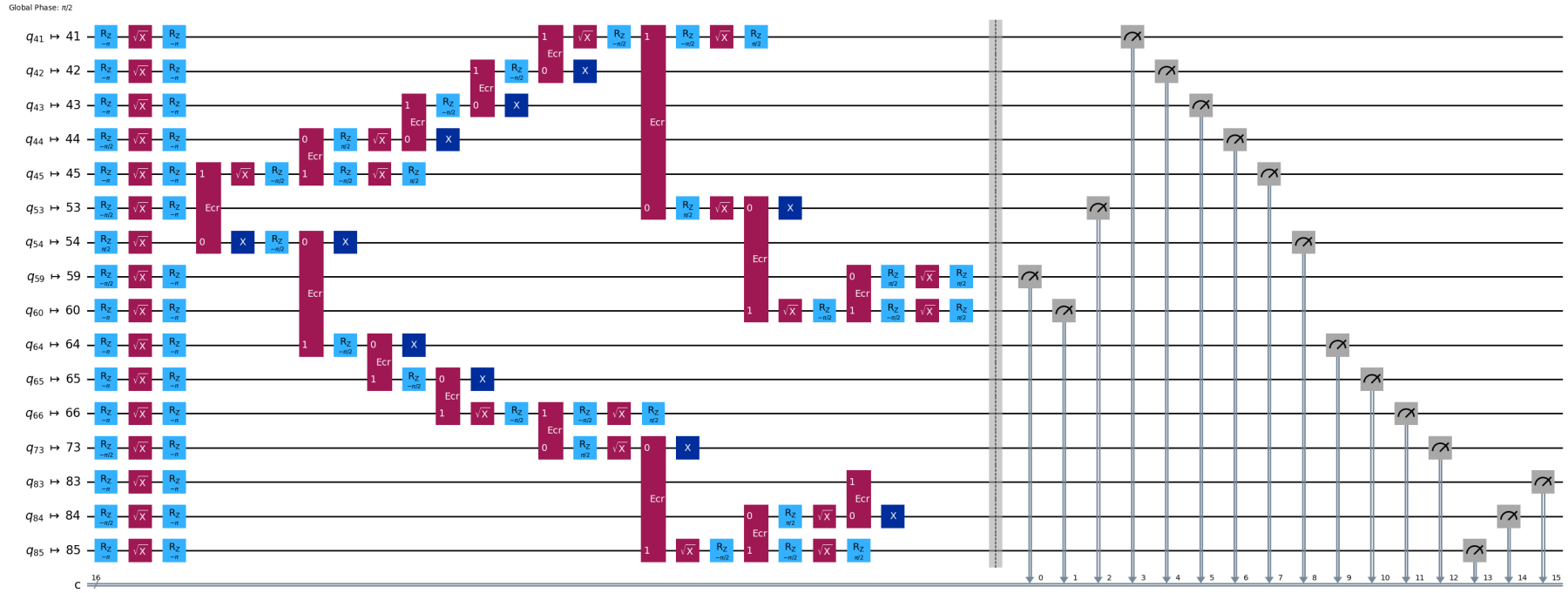


In [16]:
```
1 ghz1.depth()
```

Out[16]: 10

In [17]:
```
pm = generate_preset_pass_manager(1, backend=backend)
ghz1_transpiled = pm.run(ghz1)
ghz1_transpiled.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[17]:



In [18]:
```
print('Depth:', ghz1_transpiled.depth())
print('Two-qubit Depth:', ghz1_transpiled.depth(filter_function=lambda x: x.operation.num_qubits==2))
```

```
Depth: 27
Two-qubit Depth: 8
```

In [37]:
```
opts = SamplerOptions()
```

In [20]:
```
res = execute_ghz_fidelity(
    ghz_circuit=ghz1,
    physical_qubits=best_qubit_chain,
    backend=backend,
    sampler_options=opts,
)
```

```
Sampler job id: cv3dgs3qxmm00089yca0, shots=40000
Estimator job id: cv3dgwk12hg0008f2af0
```

In [40]:
```
job_s = service.job('cv3dgs3qxmm00089yca0') # Use your job id showed above.
job_e = service.job('cv3dgwk12hg0008f2af0')
print(job_s.status(), job_e.status())
```

```
DONE DONE
```

Be careful to execute the next cell after the above jobs statuses have become 'DONE', to show the result using the `check_ghz_fidelity_from_jobs` function.

In [41]:
```
N = 16
# Check fidelity from job IDs
res = check_ghz_fidelity_from_jobs(
    sampler_job=job_s,
    estimator_job=job_e,
    num_qubits=N,
)
```

```
N=16: |00..0>: 1163, |11..1>: 12918, |3rd>: 3835 (0000000010000000)
P(|00..0>)=0.029075, P(|11..1>)=0.32295
REM: Coherence (non-diagonal): 0.330655
GHZ fidelity = 0.341340 ± 0.003363
GME (genuinely multipartite entangled) test: Failed
```

In [ ]:
```
result = job_s.result()
plot_histogram(result[0].data.c.get_counts(), figsize=(30, 5))
```

This result does not meet the criteria. Let's move to the next idea.

Strategy 2. Balanced tree of qubits

The next idea is to find a balanced tree of qubits. Using the tree rather than the chain, the circuit depth should become lower. Before that, we remove nodes with "bad" readout errors and edges with "bad" gate errors from the coupling graph.

In [16]:
```
BAD_READOUT_ERROR_THRESHOLD = 0.1
BAD_ECRGATE_ERROR_THRESHOLD = 0.1
bad_readout_qubits = [q for q in range(backend.num_qubits) if backend.target["measure"][(q, )].error > E
bad_ecrgate_edges = [qpair for qpair in backend.target["ecr"] if backend.target["ecr"][qpair].error > BA
print("Bad readout qubits:", bad_readout_qubits)
print("Bad ECR gates:", bad_ecrgate_edges)
```

```
Bad readout qubits: [0, 1, 3, 14, 18, 28, 37, 57, 76, 90, 108, 117]
Bad ECR gates: [(76, 77), (94, 90), (106, 107), (107, 108), (112, 108), (117, 116), (118, 117)]
```

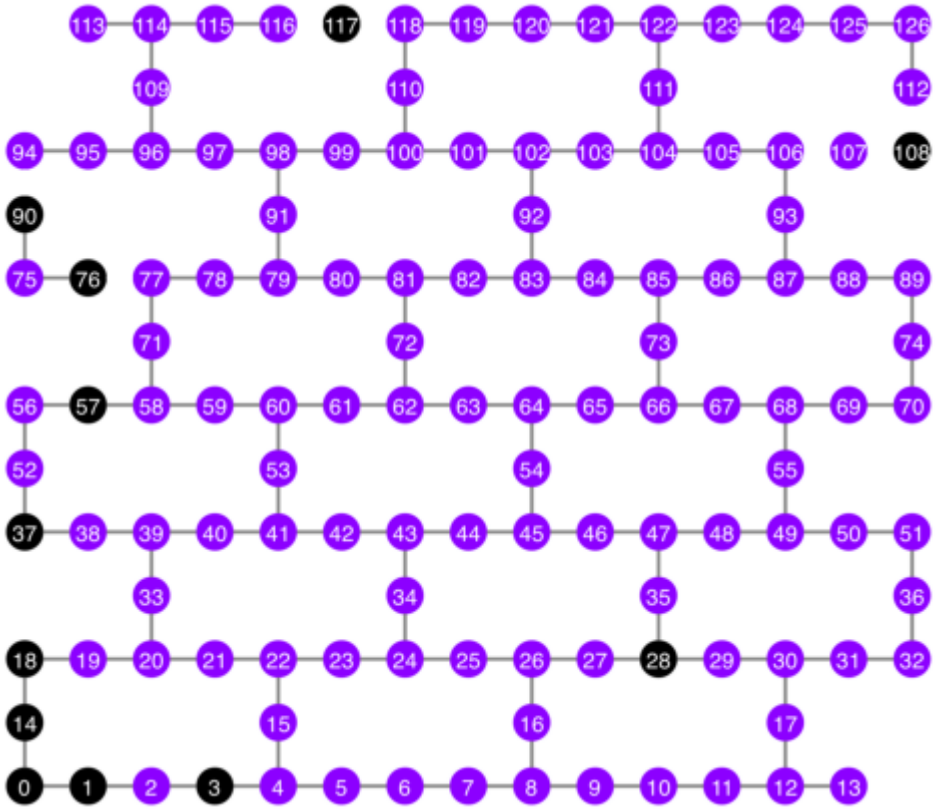
In [17]:
```
g = backend.coupling_map.graph.copy().to_undirected()
g.remove_edges_from(bad_ecrgate_edges)  # remove edge first (otherwise may fail with a NoEdgeBetweenNode
g.remove_nodes_from(bad_readout_qubits)
```

Let's draw the coupling map graph without the bad edges and bad qubits.

In [18]:

```
qubit_color = []
for i in range(133):
    if i in bad_readout_qubits:
        qubit_color.append("#000000") #black
    else:
        qubit_color.append("#8c00ff") #purple
line_color = []
for e in backend.target.build_coupling_map().get_edges():
    if e in bad_ecrgate_edges:
        line_color.append("#ffffff") #white
    else:
        line_color.append("#888888") #gray
plot_gate_map(backend, qubit_color=qubit_color, line_color=line_color, qubit_size=50, font_size=25, figs
```

Out[18]:



We try to create a 16 qubit GHZ state as before.

In [19]:
```
1 N = 16
```

We call the `betweenness_centrality` function to find a qubit for the root node. The node with the highest value of betweenness centrality is at the center of the graph. Reference: https://www.rustworkx.org/tutorial/betweenness_centrality.html (https://www.rustworkx.org/tutorial/betweenness_centrality.html)

Or you can select it manually.

In [20]:
```
1 #central = 65 #Select the center node manually
2 c_degree = dict(rx.betweenness_centrality(g))
3 central = max(c_degree, key=c_degree.get)
4 central
```

Out[20]:
```
43
```

Starting from the root node, generate a tree by breadth first search (BFS). Reference: https://qiskit.org/ecosystem/rustworkx/apiref/rustworkx.bfs_search.html#rustworkx-bfs-search (https://qiskit.org/ecosystem/rustworkx/apiref/rustworkx.bfs_search.html#rustworkx-bfs-search)

In [21]:
```
class TreeEdgesRecorder(rx.visit.BFSVisitor):
    def __init__(self, N):
        self.edges = []
        self.N = N

    def tree_edge(self, edge):
        self.edges.append(edge)
        if len(self.edges) >= self.N-1:
            raise rx.visit.StopSearch()

vis = TreeEdgesRecorder(N)
rx.bfs_search(g, [central], vis)
best_qubits = sorted(list(set(q for e in vis.edges for q in (e[0], e[1]))))
#print('Tree edges:', vis.edges)
```

In [27]:
```
print('Qubits selected:', best_qubits)
```

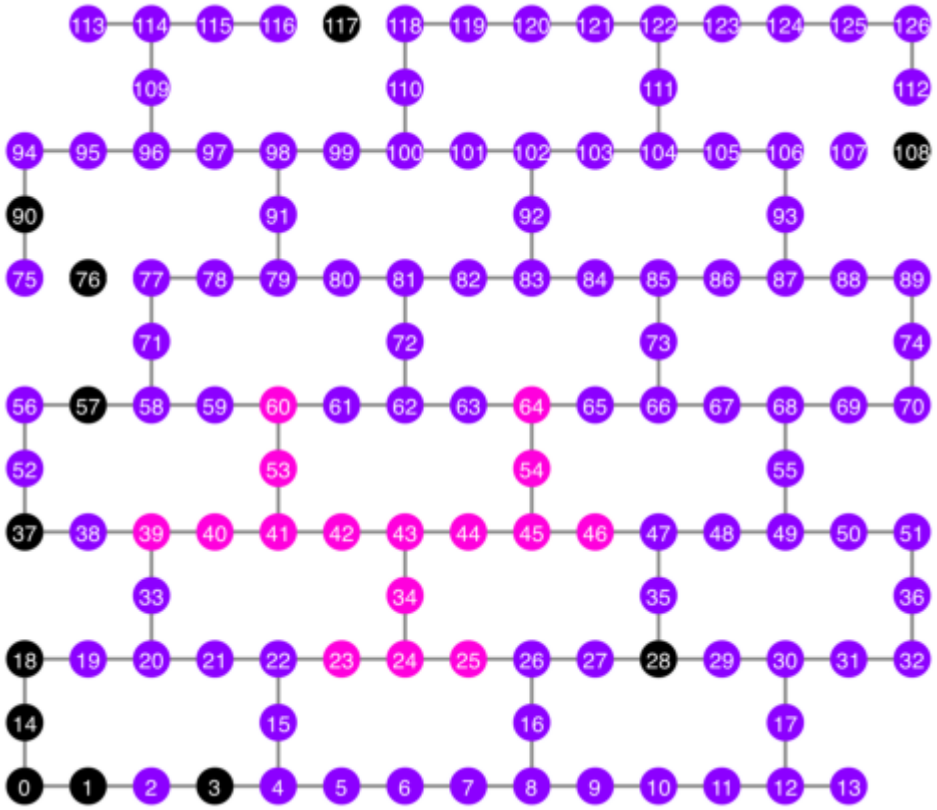
```
Qubits selected: [23, 24, 25, 34, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 60, 64]
```

Let us plot the selected qubits, shown in pink, in the coupling map diagram.

In [28]:

```
qubit_color = []
for i in range(133):
    if i in bad_readout_qubits:
        qubit_color.append("#000000") #black
    elif i in best_qubits:
        qubit_color.append("#ff00dd") #pink
    else:
        qubit_color.append("#8c00ff") #purple
plot_gate_map(backend, qubit_color=qubit_color, line_color=line_color, qubit_size=50, font_size=25, figs
```

Out[28]:

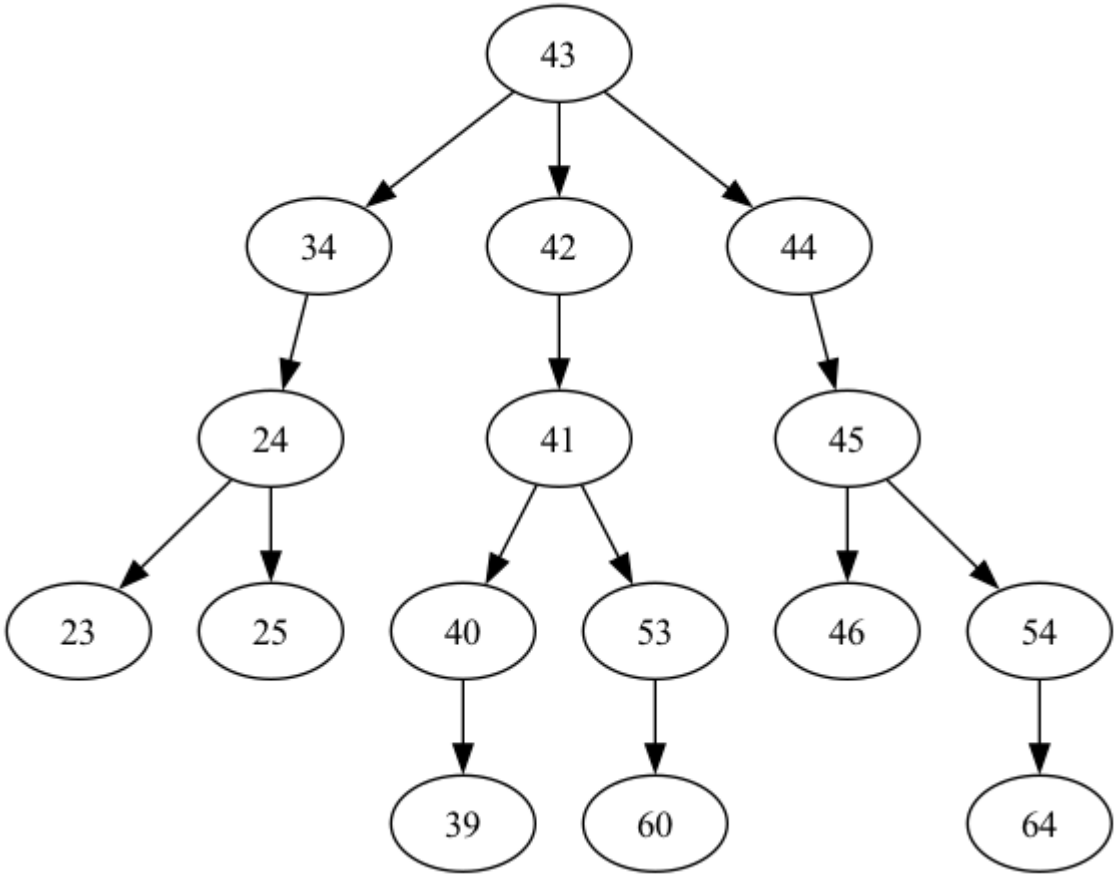


Let us show the tree structure of qubits.

In [29]:

```
from rustworkx.visualization import graphviz_draw
tree = rx.PyDiGraph()
tree.extend_from_weighted_edge_list(vis.edges)
tree.remove_nodes_from([n for n in range(max(best_qubits)+1) if n not in best_qubits])

graphviz_draw(tree, method='dot')
```

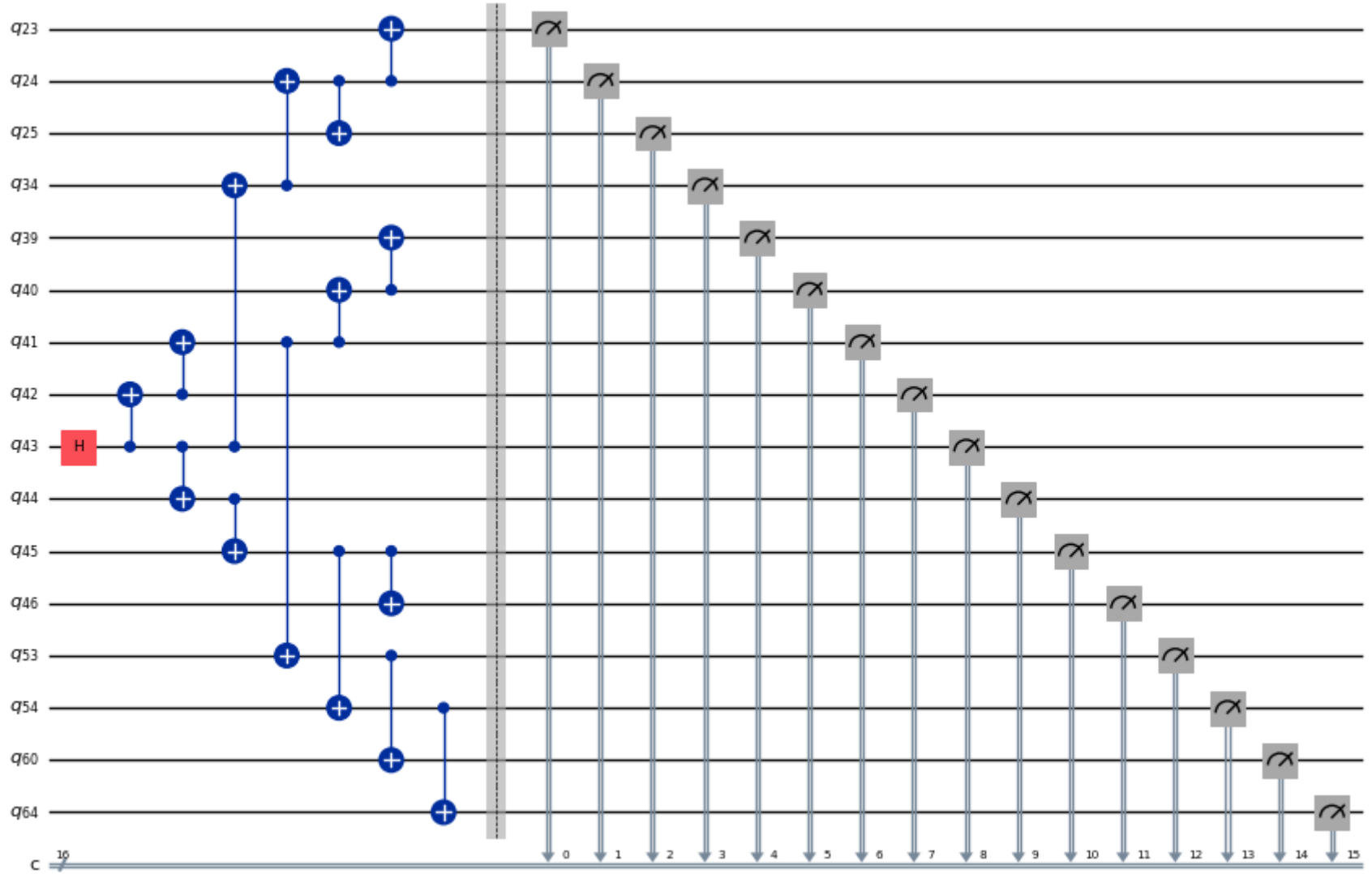
Out[29]:



In [30]:

```
ghz2 = QuantumCircuit(max(best_qubits)+1, N)

ghz2.h(tree.edge_list()[0][0]) # apply H-gate to the root node
# Apply CNOT from the root node to the each edge.
for u, v in tree.edge_list():
    ghz2.cx(u, v)
ghz2.barrier()  # for visualization
ghz2.measure(best_qubits, list(range(N)))
ghz2.draw(output="mpl", idle_wires=False, scale=0.5)
```

Out[30]:

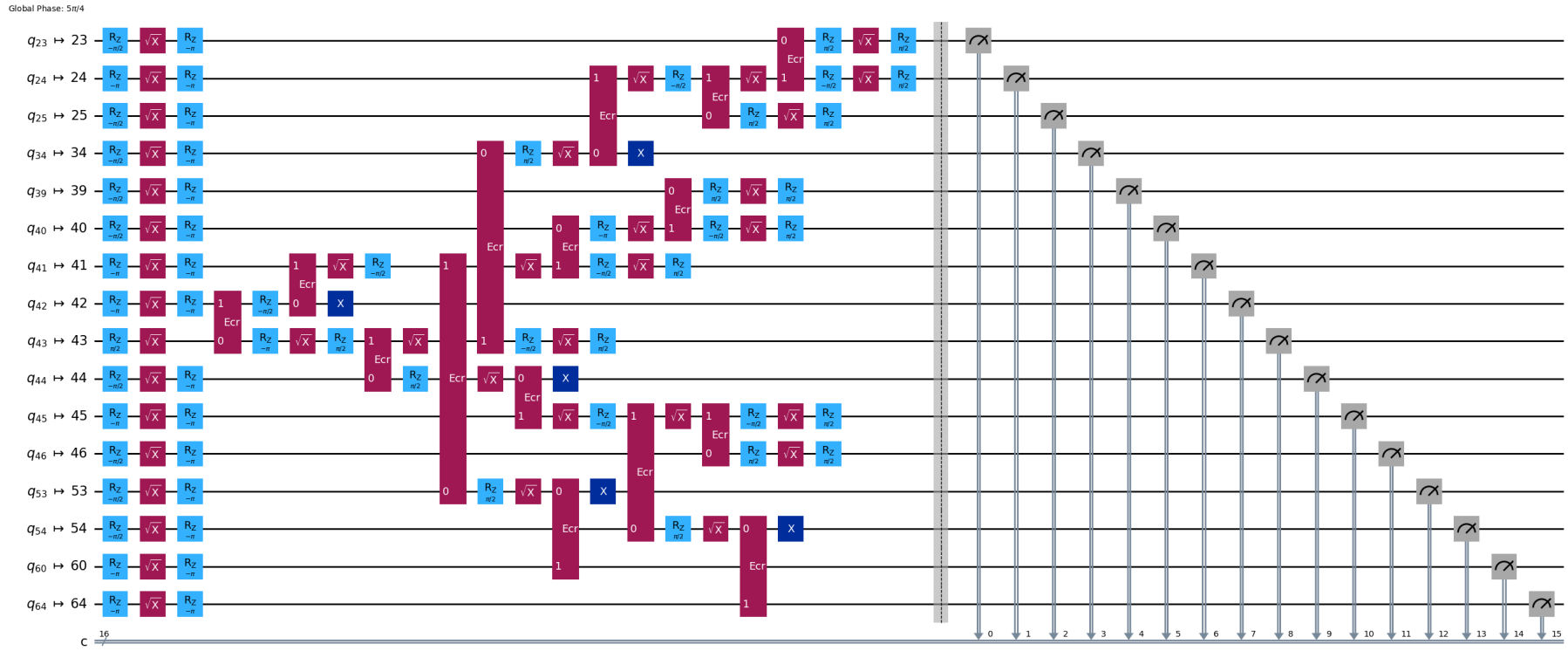


In [31]:
```
1 ghz2.depth()
```

Out[31]: 8

In [32]:
```
pm = generate_preset_pass_manager(1, backend=backend)
ghz2_transpiled = pm.run(ghz2)
ghz2_transpiled.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[32]:



In [33]:
```
print('Depth:', ghz2_transpiled.depth())
print('Two-qubit Depth:', ghz2_transpiled.depth(filter_function=lambda x: x.operation.num_qubits==2))
```

```
Depth: 22
Two-qubit Depth: 6
```

The depth of the circuit has now become much lower than that in the chain structure.

In [34]:
```
res = execute_ghz_fidelity(
    ghz_circuit=ghz2,
    physical_qubits=best_qubits,
    backend=backend,
    sampler_options=opts,
)
```

```
Sampler job id: cv3dh14fkm5g0085en3g, shots=40000
Estimator job id: cv3dh2cfkm5g0085en4g
```

In [37]:
```
job_s = service.job('cv3dh14fkm5g0085en3g') # Use your job id showed above.
job_e = service.job('cv3dh2cfkm5g0085en4g')
print(job_s.status(), job_e.status())
```

```
DONE DONE
```

In [38]:
```
N=16
# Check fidelity from job IDs
res = check_ghz_fidelity_from_jobs(
    sampler_job=job_s,
    estimator_job=job_e,
    num_qubits=N,
)
```

```
N=16: |00..0>: 713, |11..1>: 11136, |3rd>: 1192 (1111111111111011)
P(|00..0>)=0.017825, P(|11..1>)=0.2784
REM: Coherence (non-diagonal): 1.104850
GHZ fidelity = 0.700538 ± 0.009835
GME (genuinely multipartite entangled) test: Passed
```

We have successfully passed the criteria with the balanced tree structure!

In [ ]:
```
result = job_s.result()
plot_histogram(result[0].data.c.get_counts(), figsize=(30, 5))
```

Now, let us try to create a larger GHZ state, that is a 30-qubit GHZ state.

N = 30

We will follow the Qiskit Patterns (https://docs.quantum.ibm.com/guides/intro-to-patterns).

- Step 1: Map problem to quantum circuits and operators
- Step 2: Optimize for target hardware
- Step 3: Execute on target hardware
- Step 4: Post-process results

Step 1: Map problem to quantum circuits and operators and **Step 2: Optimize for target hardware**

Here we select the root node manually.

In [22]:
```
central = 62 #Select the center node manually
#c_degree = dict(rx.betweenness_centrality(g))
#central = max(c_degree, key=c_degree.get)
#central
```

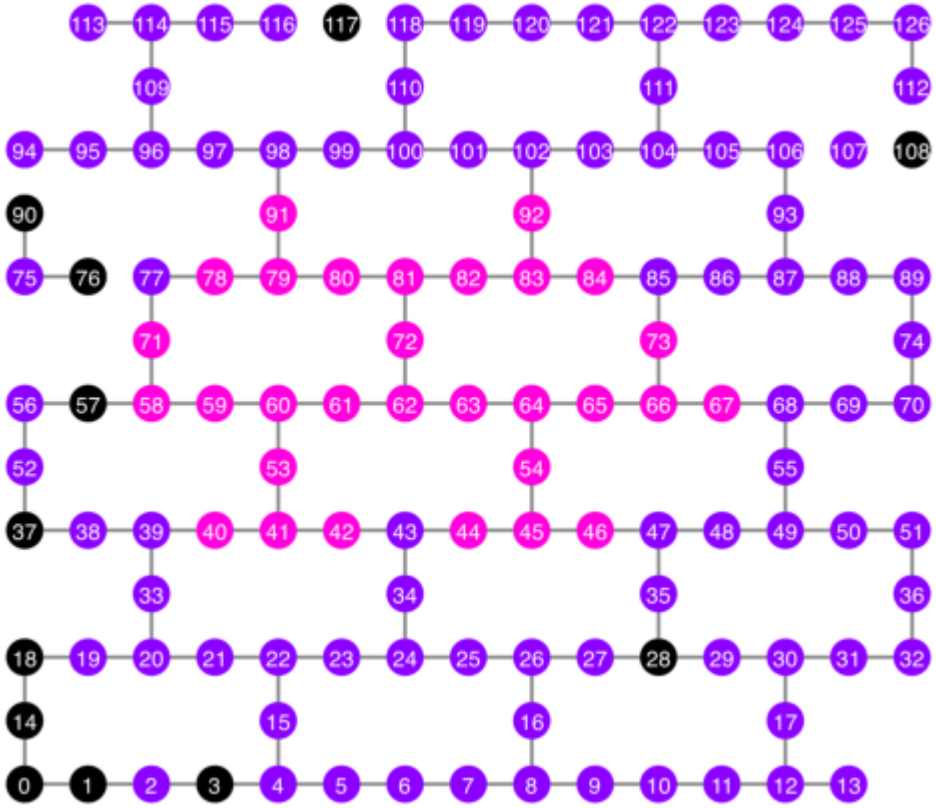
In [23]:
```
N = 30

vis = TreeEdgesRecorder(N)
rx.bfs_search(g, [central], vis)
best_qubits = sorted(list(set(q for e in vis.edges for q in (e[0], e[1]))))
print('Qubits selected:', best_qubits)
```

```
Qubits selected: [40, 41, 42, 44, 45, 46, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 78,
79, 80, 81, 82, 83, 84, 91, 92]
```

In [24]:

```
qubit_color = []
for i in range(133):
    if i in bad_readout_qubits:
        qubit_color.append("#000000")
    elif i in best_qubits:
        qubit_color.append("#ff00dd")
    else:
        qubit_color.append("#8c00ff")
line_color = []
for e in backend.target.build_coupling_map().get_edges():
    if e in bad_ecrgate_edges:
        line_color.append("#ffffff")
    else:
        line_color.append("#888888")
plot_gate_map(backend, qubit_color=qubit_color, line_color=line_color, qubit_size=50, font_size=25, figs
```

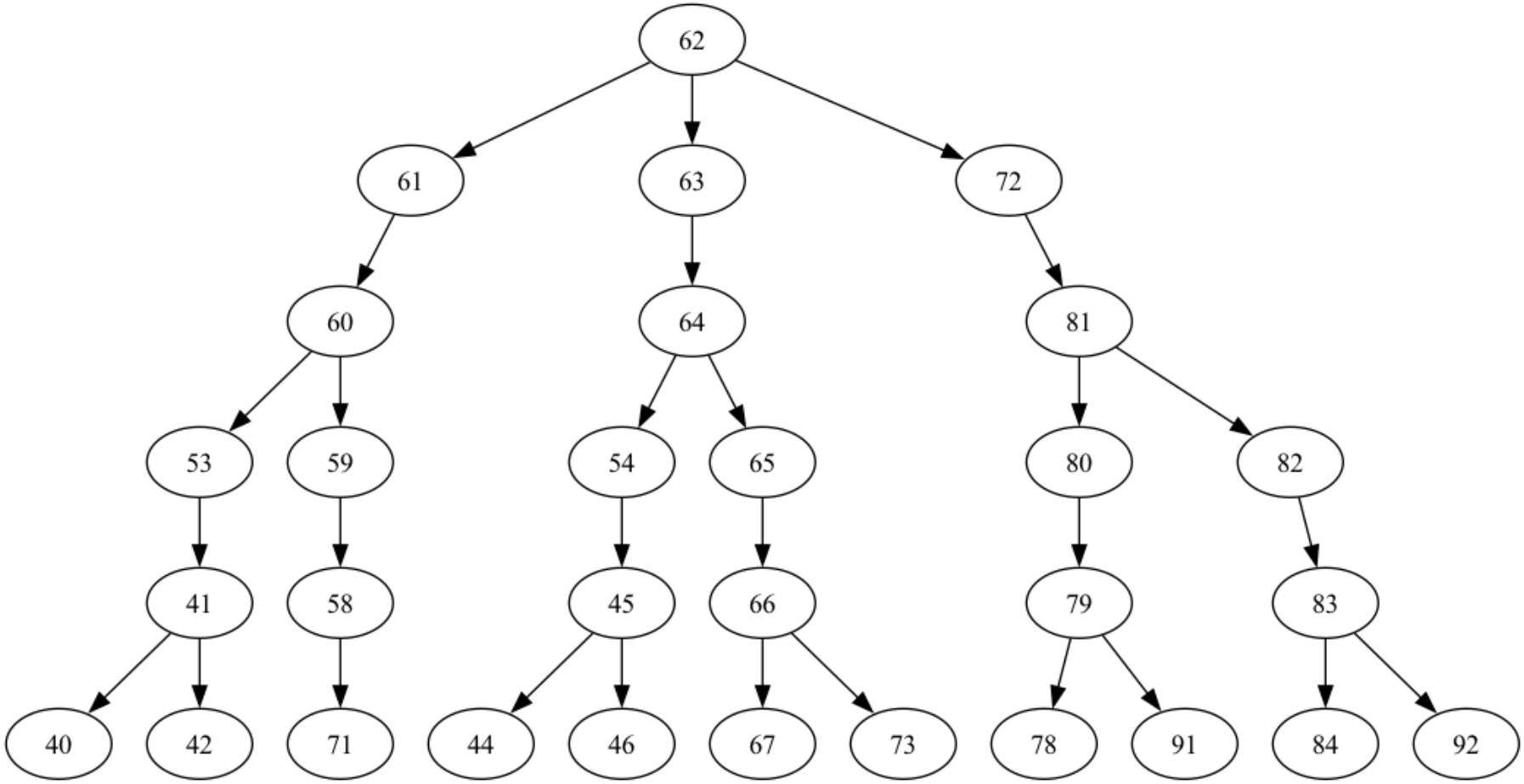
Out[24]:


113 114 115 116 117 118 119 120 121 122 123 124 125 126
109 110 111 112
94 95 96 97 98 99 100 101 102 103 104 105 106 107 108
90 91 92 93
75 76 77 78 79 80 81 82 83 84 85 86 87 88 89
71 72 73 74
56 57 58 59 60 61 62 63 64 65 66 67 68 69 70
52 53 54 55
37 38 39 40 41 42 43 44 45 46 47 48 49 50 51
33 34 35 36
18 19 20 21 22 23 24 25 26 27 28 29 30 31 32
14 15 16 17
0 1 2 3 4 5 6 7 8 9 10 11 12 13

In [25]:

```
from rustworkx.visualization import graphviz_draw
tree = rx.PyDiGraph()
tree.extend_from_weighted_edge_list(vis.edges)
tree.remove_nodes_from([n for n in range(max(best_qubits)+1) if n not in best_qubits])

graphviz_draw(tree, method='dot')
```

Out[25]:

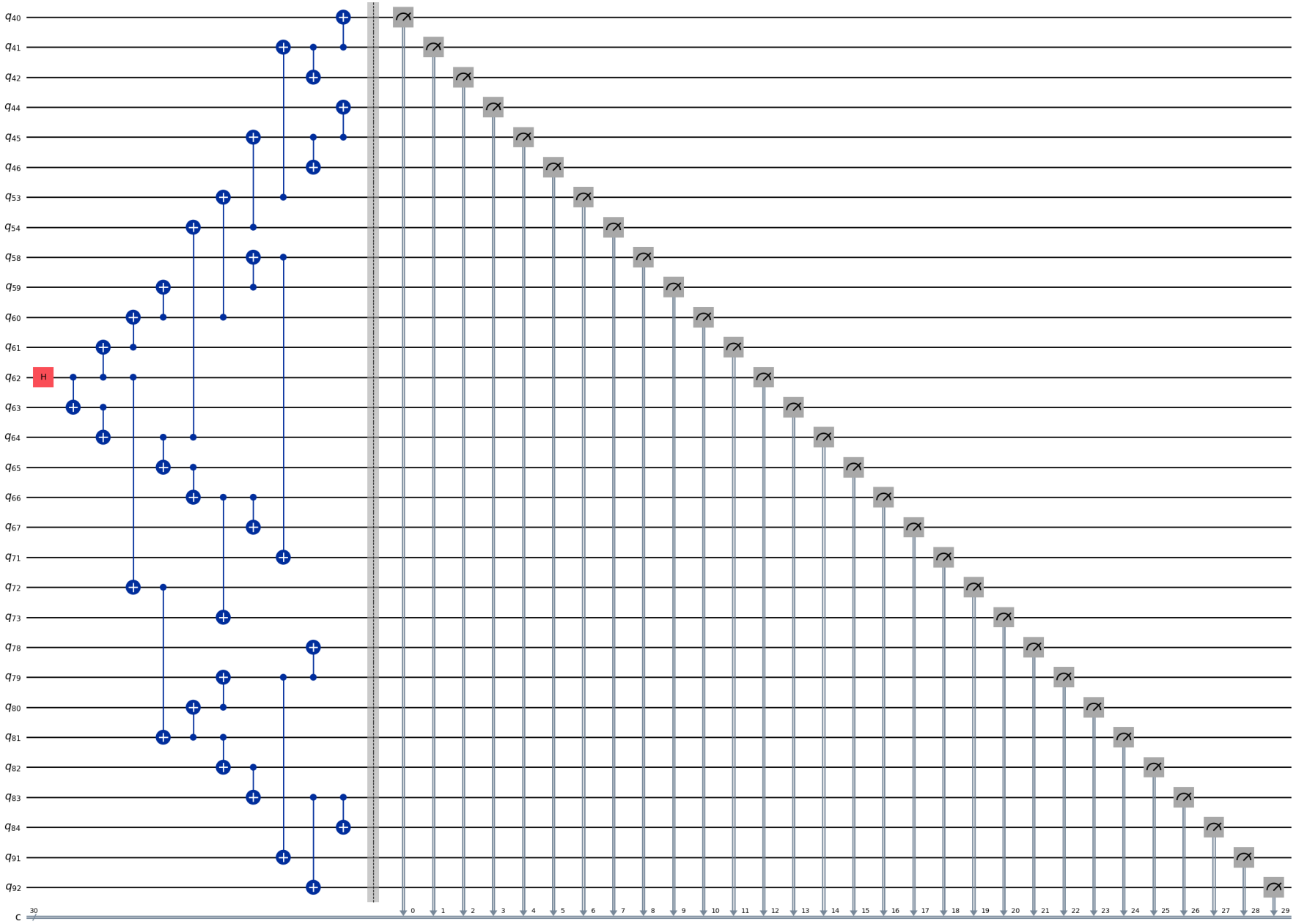


The depth of this tree is 5.

In [26]:

```
ghz3 = QuantumCircuit(max(best_qubits)+1, N)

ghz3.h(tree.edge_list()[0][0]) # apply H-gate to the root node
# Apply CNOT from the root node to the each edge.
for u, v in tree.edge_list():
    ghz3.cx(u, v)
ghz3.barrier()  # for visualization
ghz3.measure(best_qubits, list(range(N)))
ghz3.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[26]:

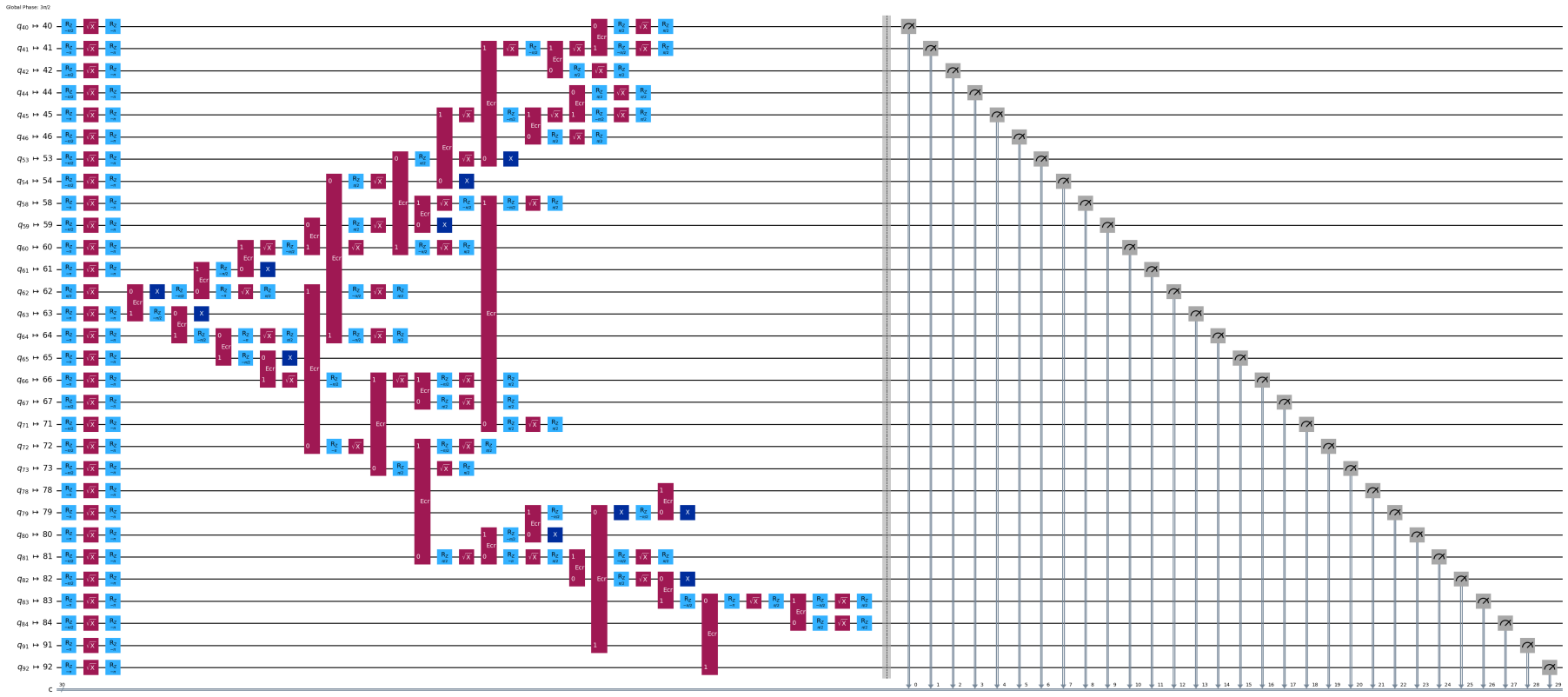

q40
q41
q42
q44
q45
q46
q53
q54
q58
q59
q60
q61
q62
H
q63
q64
q65
q66
q67
q71
q72
q73
q78
q79
q80
q81
q82
q83
q84
q91
q92
c
30
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

In [27]:
```
1 ghz3.depth()
```

Out[27]: 11

In [28]:
```
1 pm = generate_preset_pass_manager(1, backend=backend)
2 ghz3_transpiled = pm.run(ghz3)
3 ghz3_transpiled.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[28]:



In [29]:
```
1 print('Depth:', ghz3_transpiled.depth())
2 print('Two-qubit Depth:', ghz3_transpiled.depth(filter_function=lambda x: x.operation.num_qubits==2))
```

```
Depth: 34
Two-qubit Depth: 9
```

Select a different root node manually

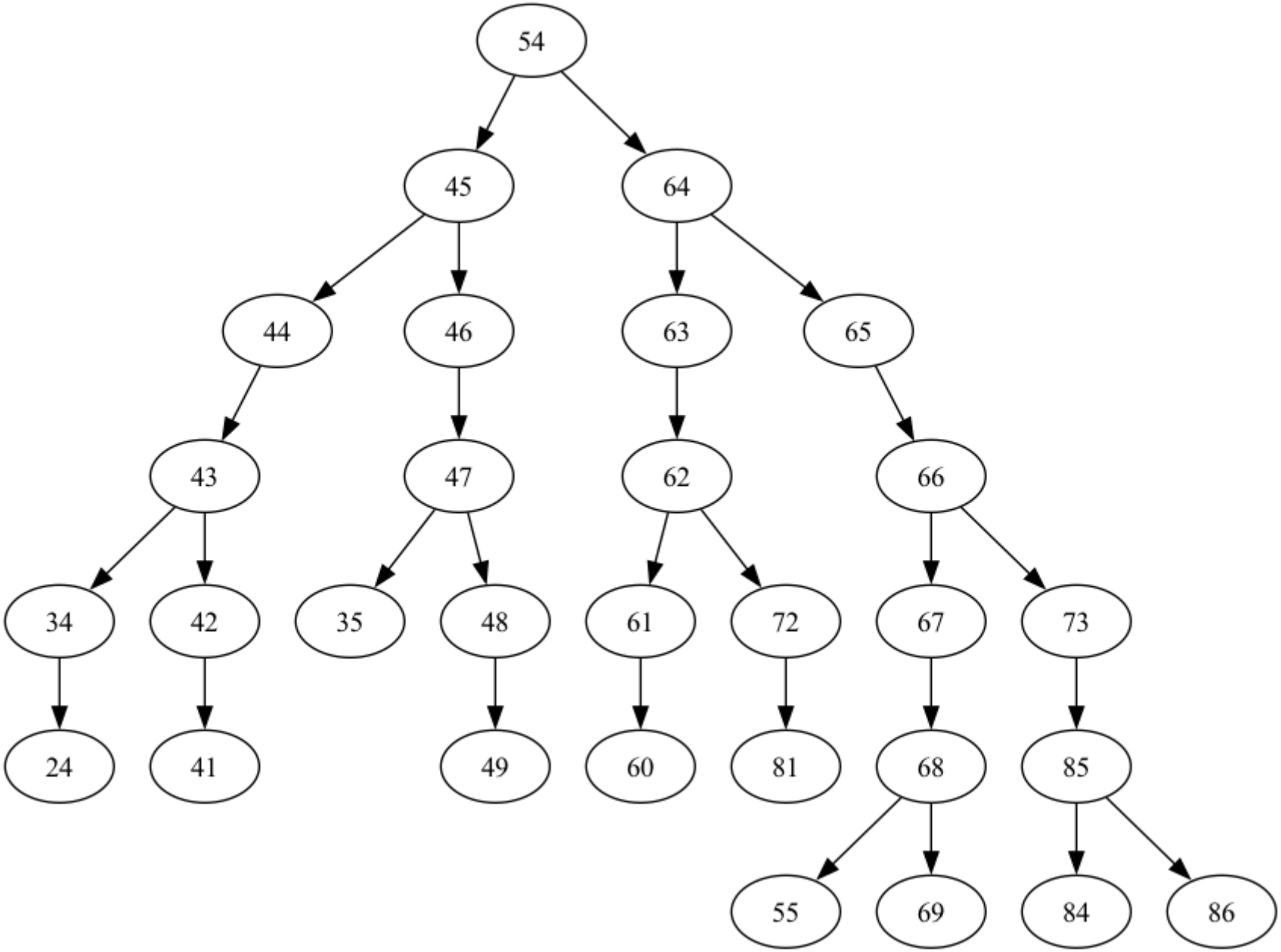
In [30]:
```
central = 54

vis = TreeEdgesRecorder(N)
rx.bfs_search(g, [central], vis)
best_qubits = sorted(list(set(q for e in vis.edges for q in (e[0], e[1]))))
print('Qubits selected:', best_qubits)
```

```
Qubits selected: [24, 34, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 55, 60, 61, 62, 63, 64, 65, 66, 67,
68, 69, 72, 73, 81, 84, 85, 86]
```

In [31]:

```
from rustworkx.visualization import graphviz_draw
tree = rx.PyDiGraph()
tree.extend_from_weighted_edge_list(vis.edges)
tree.remove_nodes_from([n for n in range(max(best_qubits)+1) if n not in best_qubits])

graphviz_draw(tree, method='dot')
```

Out[31]:

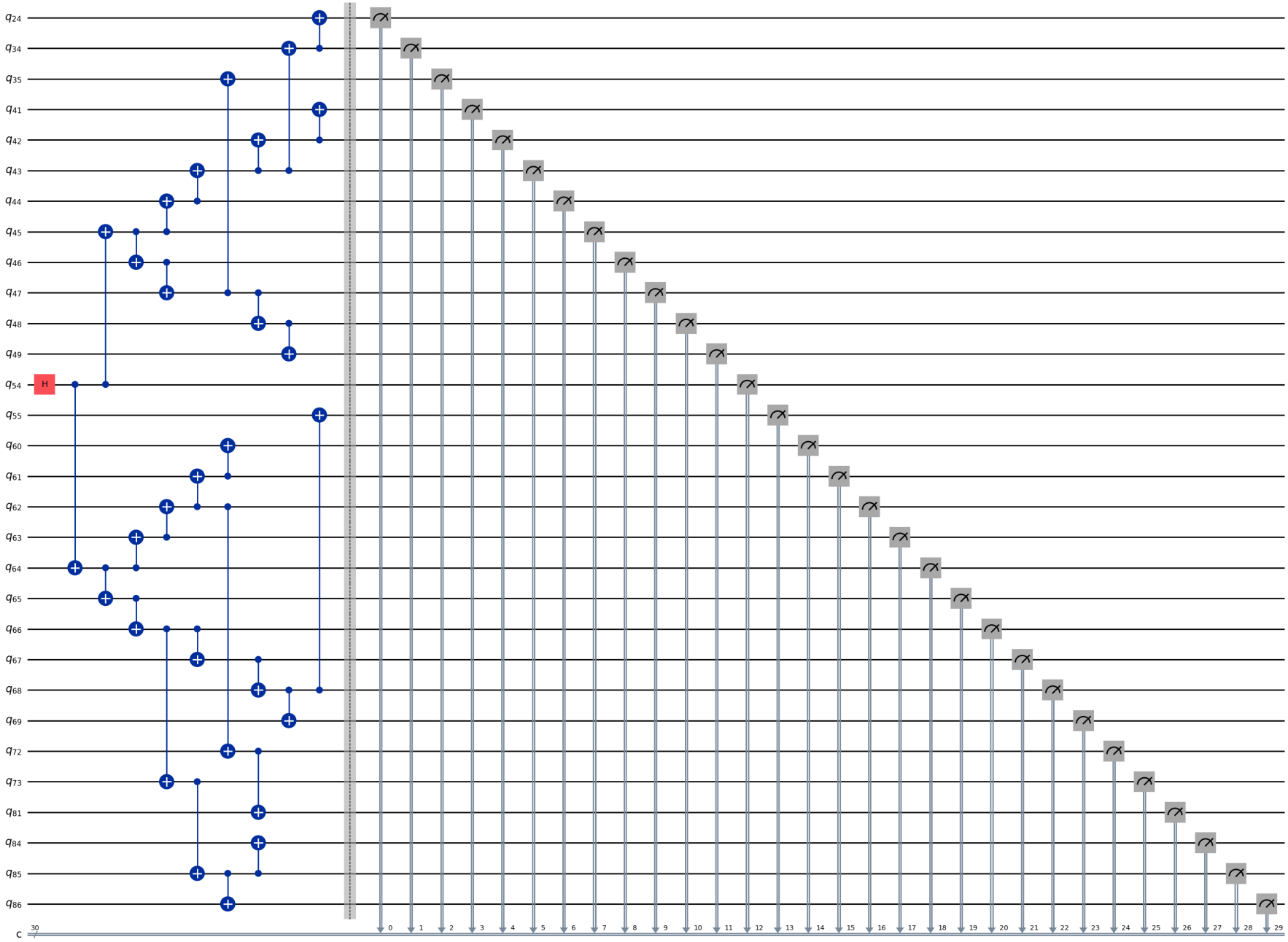


The depth of this tree is 6.

In [32]:

```
ghz3 = QuantumCircuit(max(best_qubits)+1, N)

ghz3.h(tree.edge_list()[0][0]) # apply H-gate to the root node
# Apply CNOT from the root node to the each edge.
for u, v in tree.edge_list():
    ghz3.cx(u, v)
ghz3.barrier()  # for visualization
ghz3.measure(best_qubits, list(range(N)))
ghz3.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[32]:

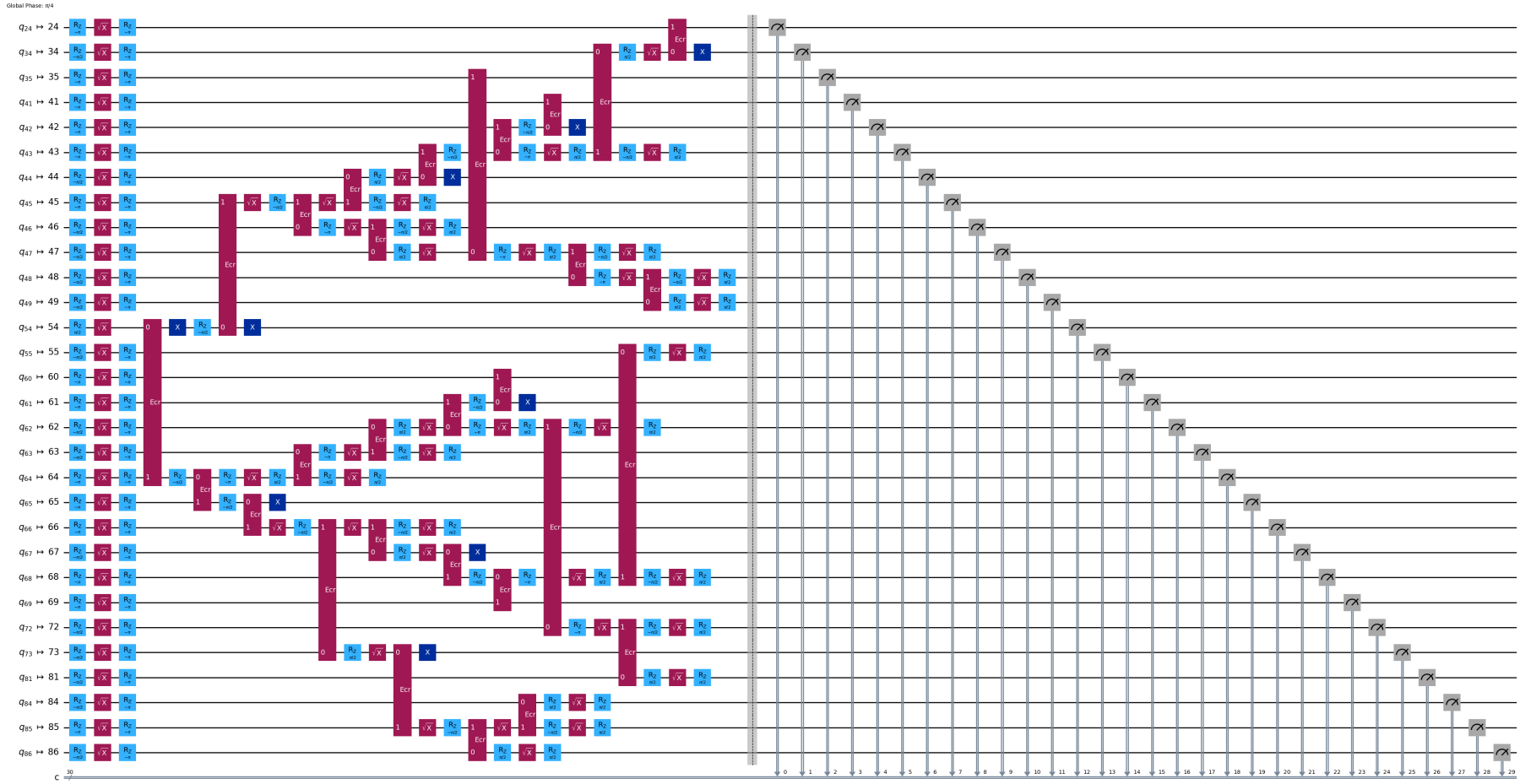

q24
q34
q35
q41
q42
q43
q44
q45
q46
q47
q48
q49
q54
H
q55
q60
q61
q62
q63
q64
q65
q66
q67
q68
q69
q72
q73
q81
q84
q85
q86
c
30
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

In [33]:
```
1 ghz3.depth()
```

Out[33]: 10

In [34]:
```
1 pm = generate_preset_pass_manager(1, backend=backend)
2 ghz3_transpiled = pm.run(ghz3)
3 ghz3_transpiled.draw(output="mpl", idle_wires=False, fold=-1)
```

Out[34]:



In [35]:
```
print('Depth:', ghz3_transpiled.depth())
print('Two-qubit Depth:', ghz3_transpiled.depth(filter_function=lambda x: x.operation.num_qubits==2))
```

```
Depth: 27
Two-qubit Depth: 8
```

Surprisingly, while the tree depth increased from 5 to 6, the two-qubit depth decreased from 9 to 8! So let us use the latter circuit.

Step 3: Execute on target hardware

In [38]:
```
res = execute_ghz_fidelity(
    ghz_circuit=ghz3,
    physical_qubits=best_qubits,
    backend=backend,
    sampler_options=opts,
)
```

```
Sampler job id: cv3ywhjx55b0008d33wg, shots=40000
Estimator job id: cv3ywjtx55b0008d33xg
```

In [41]:
```
job_s = service.job('cv3ywhjx55b0008d33wg') # Use your job id showed above.
job_e = service.job('cv3ywjtx55b0008d33xg')
print(job_s.status(), job_e.status())
```

```
DONE DONE
```

Step 4: Post-process results

In [42]:
```
N=30
# Check fidelity from job IDs
res = check_ghz_fidelity_from_jobs(
    sampler_job=job_s,
    estimator_job=job_e,
    num_qubits=N,
)
```

```
N=30: |00..0>: 0, |11..1>: 1821, |3rd>: 676 (00000000010110000000010000000)
P(|00..0>)=0.0, P(|11..1>)=0.045525
REM: Coherence (non-diagonal): 0.478084
GHZ fidelity = 0.261805 ± 0.011188
GME (genuinely multipartite entangled) test: Failed
```

As you can see, this result has not met the criteria.

In [ ]:
```
# It will take some time
result = job_s.result()
plot_histogram(result[0].data.c.get_counts(), figsize=(30, 5))
```

Strategy 3. Run with the error suppression options

You can set the error suppression options in Sampler V2. Please refer https://docs.quantum.ibm.com/guides/configure-error-mitigation#advanced-error (https://docs.quantum.ibm.com/guides/configure-error-mitigation#advanced-error), and https://docs.quantum.ibm.com/api/qiskit-ibm-runtime/dev/qiskit_ibm_runtime.options.ExecutionOptionsV2 (https://docs.quantum.ibm.com/api/qiskit-ibm-runtime/dev/qiskit_ibm_runtime.options.ExecutionOptionsV2)

In [39]:
```
opts = SamplerOptions()
opts.dynamical_decoupling.enable = True
opts.execution.rep_delay = 0.0005
opts.twirling.enable_gates = True
```

In [40]:
```
res = execute_ghz_fidelity(
    ghz_circuit=ghz3,
    physical_qubits=best_qubits,
    backend=backend,
    sampler_options=opts,
)
```

```
Sampler job id: cv3ywvkfkm5g008xxe2g, shots=40000
Estimator job id: cv3ywzbqxmm0008vrzt0
```

In [44]:
```
job_s = service.job('cv3ywvkfkm5g008xxe2g') # Use your job id showed above.
job_e = service.job('cv3ywzbqxmm0008vrzt0')
print(job_s.status(), job_e.status())
```

```
DONE DONE
```

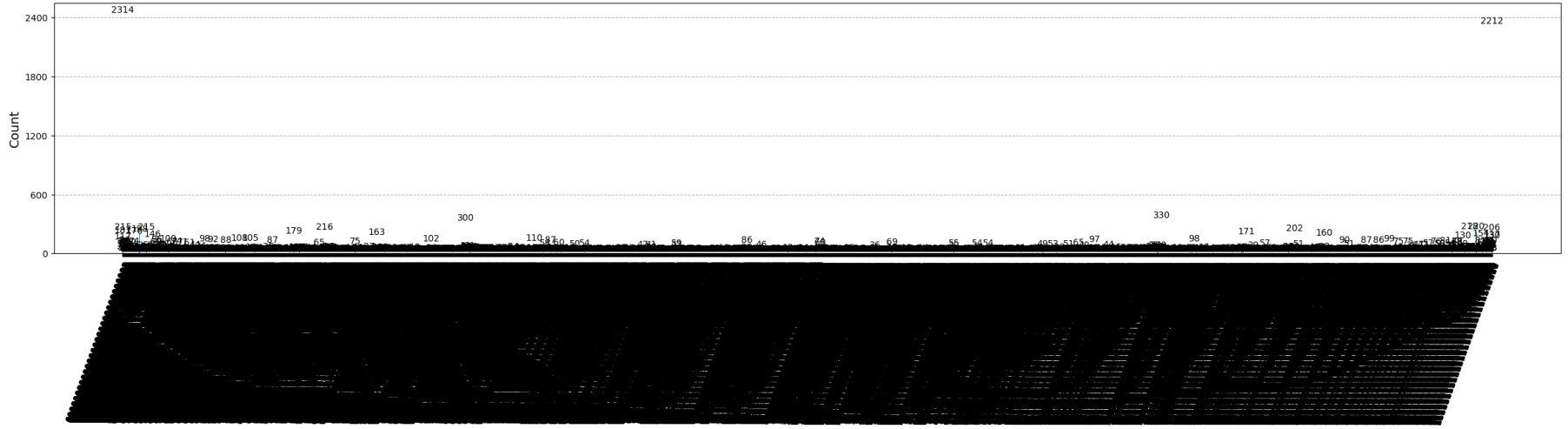
In [45]:
```
N = 30
```

In [46]:
```
# Check fidelity from job IDs
res = check_ghz_fidelity_from_jobs(
    sampler_job=job_s,
    estimator_job=job_e,
    num_qubits=N,
)
```

```
N=30: |00..0>: 2314, |11..1>: 2212, |3rd>: 330 (11101111111111111111111111111111)
P(|00..0>)=0.05785, P(|11..1>)=0.0553
REM: Coherence (non-diagonal): 0.492400
GHZ fidelity = 0.302775 ± 0.011243
GME (genuinely multipartite entangled) test: Failed
```

In [47]:
```
# It will take some time
result = job_s.result()
plot_histogram(result[0].data.c.get_counts(), figsize=(30, 5))
```

Out[47]:



The result has improved but still has not met the criteria.

We have seen three ideas so far. You can combine and expand these ideas or you come up with your own ideas to create a better GHZ circuit. Now let's review the goal again.

Your goal (Recap)

Build a GHZ circuit for 20 qubits or more so that the measurement result meets the criteria: **The fidelity of your GHZ state > 0.5.**

- You need to use an Eagle device (`ibm_kyiv` , etc.) and set the shots number as 40,000.
- You should execute the GHZ circuit using the `execute_ghz_fidelity` function, and calculate the fidelity using the `check_ghz_fidelity_from_jobs` function.

You need to find the biggest qubits - GHZ circuit which meet the criteria. Write your code below, show the result with the function `check_ghz_fidelity_from_jobs` , and submit this notebook.

Continued in Part 2.