In [1]:
```
import qiskit

qiskit.__version__
```

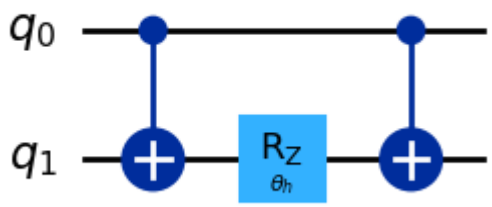
Out[1]: '1.4.2'

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np
import rustworkx as rx

from qiskit import QuantumCircuit, transpile
from qiskit.circuit import Parameter
from qiskit.circuit.library import YGate
from qiskit.quantum_info import SparsePauliOp
from qiskit_ibm_runtime import QiskitRuntimeService, fake_provider, EstimatorV2 as Estimator
from qiskit_aer import AerSimulator
```

In [3]:
```
service = QiskitRuntimeService()
```

In [5]:
```
from qiskit.circuit.library import RZZGate
θ_h = Parameter("$\\theta_h$")
qc1= QuantumCircuit(2)
qc1.append(RZZGate(θ_h),[0, 1])
qc1.decompose(reps=1).draw("mpl")
```

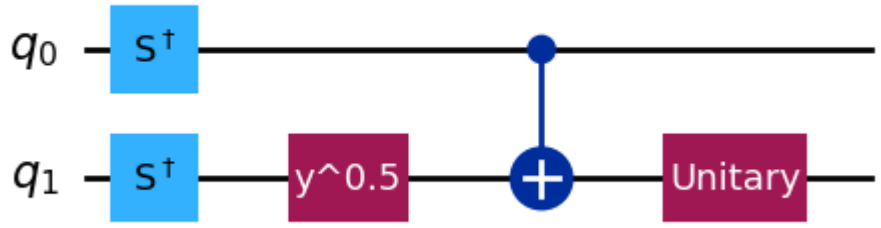
Out[5]:



In [6]:

```
qc2 = QuantumCircuit(2)

qc2.sdg([0, 1])
qc2.append(YGate().power(1/2), [1])
qc2.cx(0, 1)
qc2.append(YGate().power(1/2).adjoint(), [1])

qc2.draw("mpl")
```

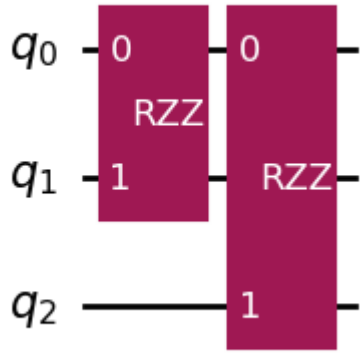
Out[6]:



In [7]:

```
rzz = qc2.to_gate(label="RZZ")
```

In [8]:
```
qc3 = QuantumCircuit(3)
qc3.append(rzz, [0,1])
qc3.append(rzz, [0,2])
display(qc3.draw("mpl"))
#display(qc.decompose(reps=1).draw("mpl"))
```



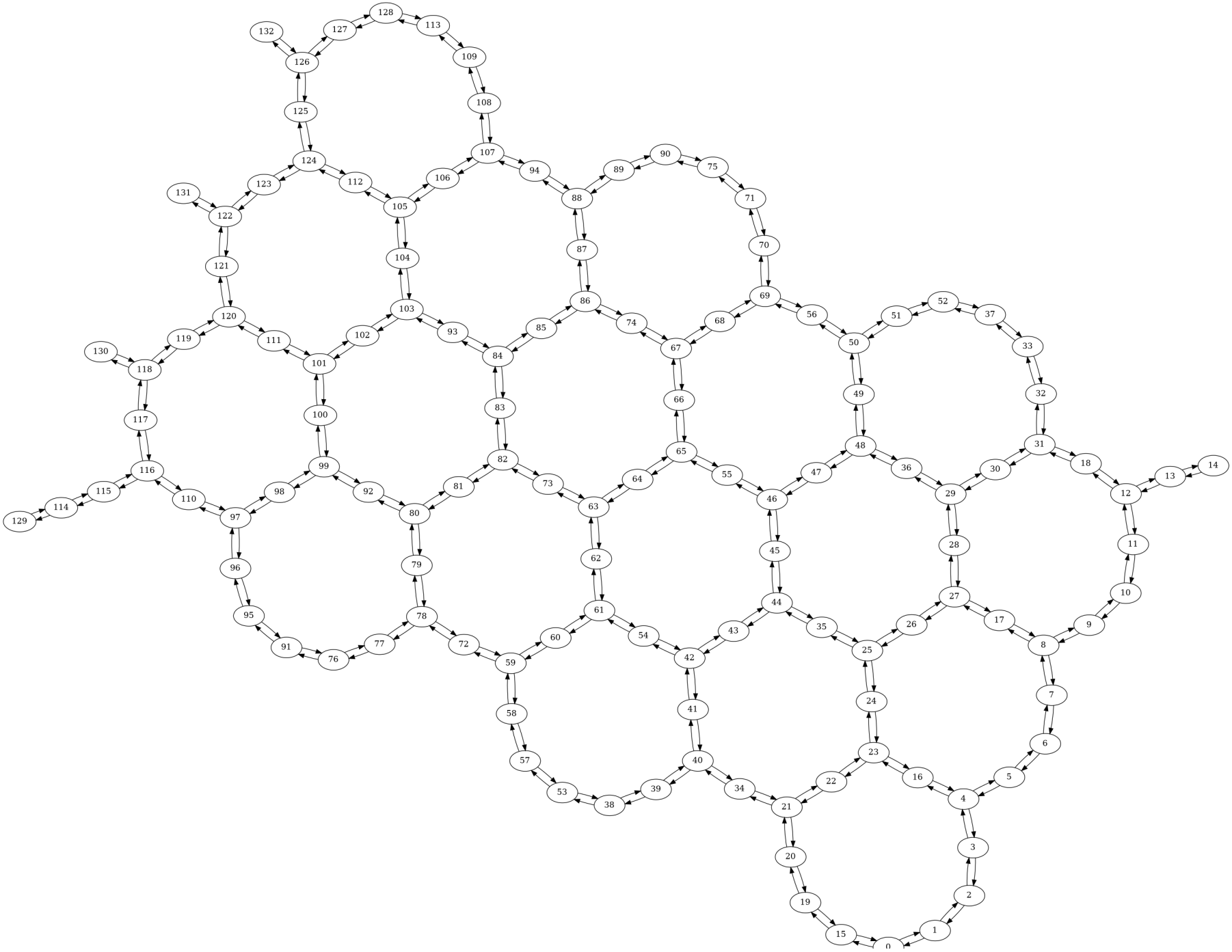
In [9]:
```
from qiskit.quantum_info import Operator

op1 = Operator(qc1.assign_parameters([-np.pi/2]))
op2 = Operator(qc2)

op1.equiv(op2)
```

Out[9]: True

In [11]:

```
backend = service.backend("ibm_torino")
backend.coupling_map.draw()
```

Out[11]: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2
19
15
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0

In [12]:
```python
def color_coupling_map(backend):
    graph = backend.coupling_map.graph
    undirected_graph = graph.to_undirected(multigraph=False)
    edge_color_map = rx.graph_bipartite_edge_color(undirected_graph)
    if (edge_color_map is None):
        edge_color_map = rx.graph_greedy_edge_color(undirected_graph)
    # build a map from color to a list of edges
    edge_index_map = undirected_graph.edge_index_map()
    color_edges_map = {color: [] for color in edge_color_map.values()}
    for edge_index, color in edge_color_map.items():
        color_edges_map[color].append((edge_index_map[edge_index][0], edge_index_map[edge_index][1]))
    return edge_color_map, color_edges_map
```

In [13]:
```python
edge_color_map, color_edges_map = color_coupling_map(backend)
print(f"{backend.name}, {backend.num_qubits}-qubit device, {len(color_edges_map.keys())} colors assigne
```

```
ibm_torino, 133-qubit device, 3 colors assigned.
```

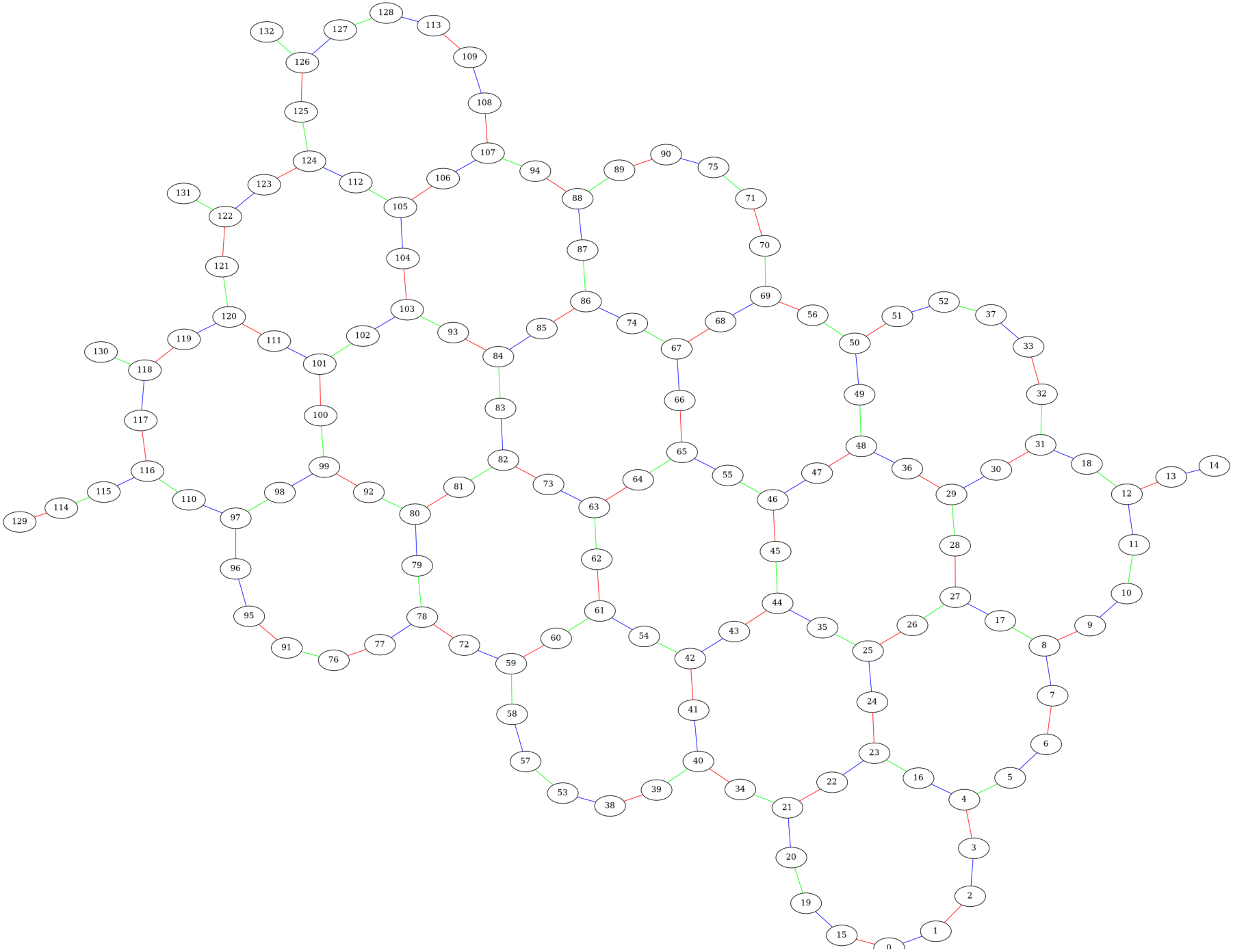
In [14]:

```
color_str_map = {0: "green", 1: "red", 2: "blue"}

undirected_graph = backend.coupling_map.graph.to_undirected(multigraph=False)
for i in undirected_graph.edge_indices():
    undirected_graph.get_edge_data_by_index(i)["color"] = color_str_map[edge_color_map[i]]

rx.visualization.graphviz_draw(undirected_graph, method="neato", edge_attr_fn=lambda edge: {"color": ed
```

Out[14]: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3
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0

In [15]:

```python
def get_utility_circuit(backend, num_steps: int, barrier: bool = False):
    num_qubits = backend.num_qubits
    _, color_edges_map = color_coupling_map(backend)
    θ_h = Parameter("$\\theta_h$")
    qc = QuantumCircuit(num_qubits)

    for i in range(num_steps):
        qc.rx(θ_h, range(num_qubits))


        for _, edge_list in color_edges_map.items():
            for edge in edge_list:
                qc.append(rzz, edge)


        if barrier:
            qc.barrier()
    return qc
```

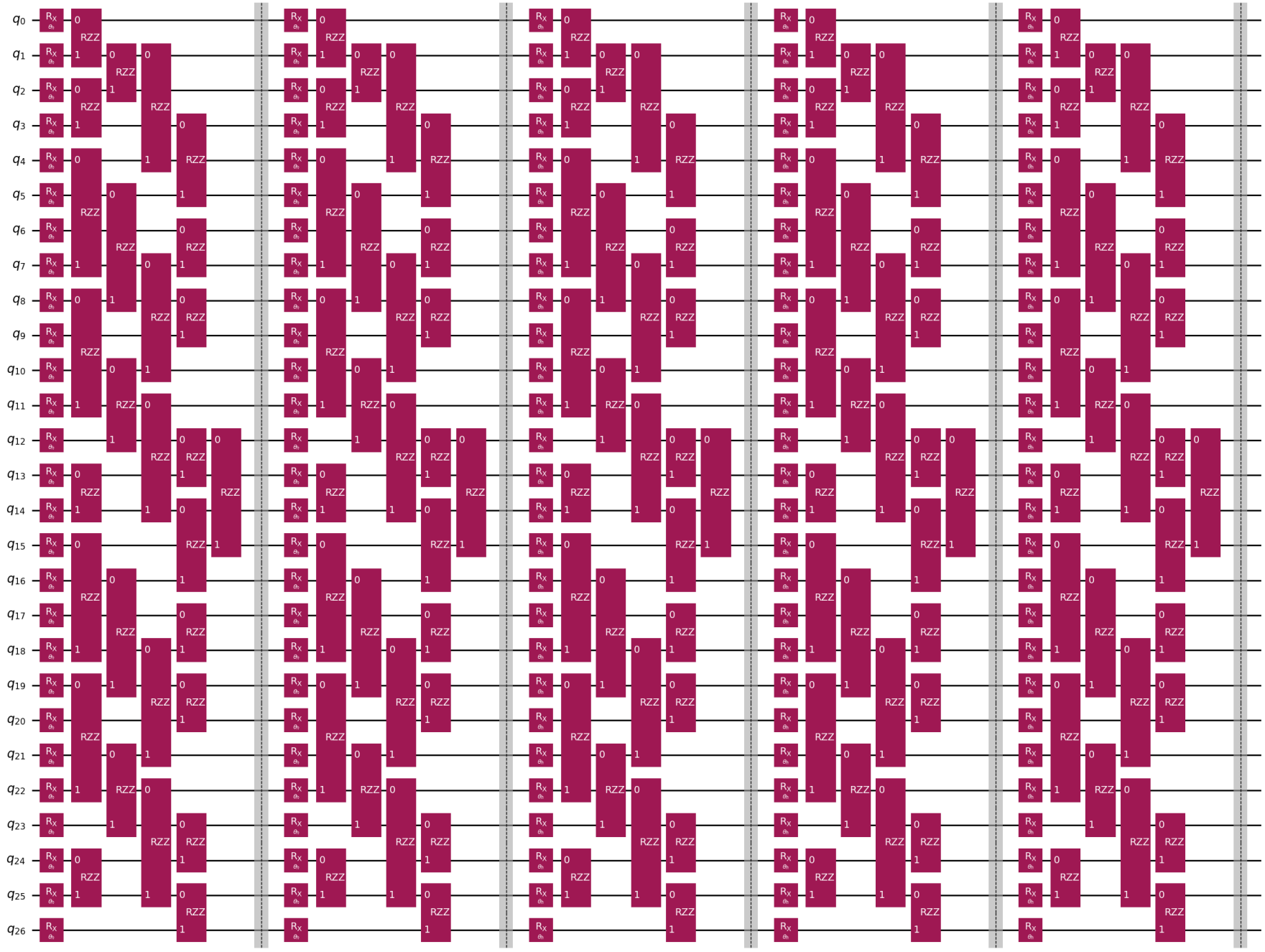
In [16]:

```python
def get_circuit_info(qc: QuantumCircuit, reps: int = 0):
    qc0 = qc.decompose(reps=reps)
    return f"{qc0.num_qubits} qubits × {qc0.depth(lambda x: x.operation.num_qubits == 2)} layers ({qc0.c
    ",  " + f"""Gate breakdown: {", ".join([f"{k.upper()} {v}" for k, v in qc0.count_ops().items()])}"""
```

In [17]:

```
backend = fake_provider.FakeTorontoV2()
num_steps = 5
qc = get_utility_circuit(backend, num_steps, True)

display(qc.draw(output="mpl", fold=-1))
print(get_circuit_info(qc, reps=0))
print(get_circuit_info(qc, reps=1))
```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q_{19}
q_{20}
q_{21}
q_{22}
q_{23}
q_{24}
q_{25}
q_{26}
RX
RZZ

```
27 qubits × 15 layers (20-depth),  Gate breakdown: CIRCUIT-168 140, RX 135, BARRIER 5
27 qubits × 15 layers (60-depth),  Gate breakdown: SDG 280, UNITARY 280, CX 140, R 135, BARRIER 5
```

In [18]:
```
backend = fake_provider.FakeTorontoV2()    # a 27 qubit fake device.
num_steps = 2
qc = get_utility_circuit(backend, num_steps)
obs = SparsePauliOp.from_sparse_list([("Z", [13], 1)], num_qubits=backend.num_qubits) # Falcon
angles = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, np.pi/2]   # We try 11 angles for theta_h.
```

In [19]:
```
backend_sim = AerSimulator()
transpiled_qc_sim = transpile(qc, backend_sim, optimization_level=1, layout_method="trivial")
transpiled_obs_sim = obs.apply_layout(layout=transpiled_qc_sim.layout)

print(get_circuit_info(qc, reps=1))
print(get_circuit_info(transpiled_qc_sim, reps=1))
```

```
27 qubits × 6 layers (23-depth),  Gate breakdown: SDG 112, UNITARY 112, CX 56, R 54
27 qubits × 6 layers (16-depth),  Gate breakdown: U3 80, CX 56, R 54, U1 32, U 28
```

In [20]:
```
%%time
params = [[p] for p in angles]
estimator = Estimator(mode=backend_sim)
pub = (transpiled_qc_sim, transpiled_obs_sim, params)
result_sim = estimator.run([pub]).result()
```

```
CPU times: user 382 ms, sys: 16.7 ms, total: 399 ms
Wall time: 435 ms
```

In [21]:
```
backend_fake = fake_provider.FakeTorontoV2()
transpiled_qc_fake = transpile(qc, backend_fake, optimization_level=1, layout_method="trivial")
transpiled_obs_fake = obs.apply_layout(layout=transpiled_qc_fake.layout)

print(get_circuit_info(qc, reps=1))
print(get_circuit_info(transpiled_qc_fake, reps=1))
```

```
27 qubits × 6 layers (23-depth),  Gate breakdown: SDG 112, UNITARY 112, CX 56, R 54
27 qubits × 6 layers (49-depth),  Gate breakdown: SDG 324, U1 274, H 162, CX 56, U3 14
```

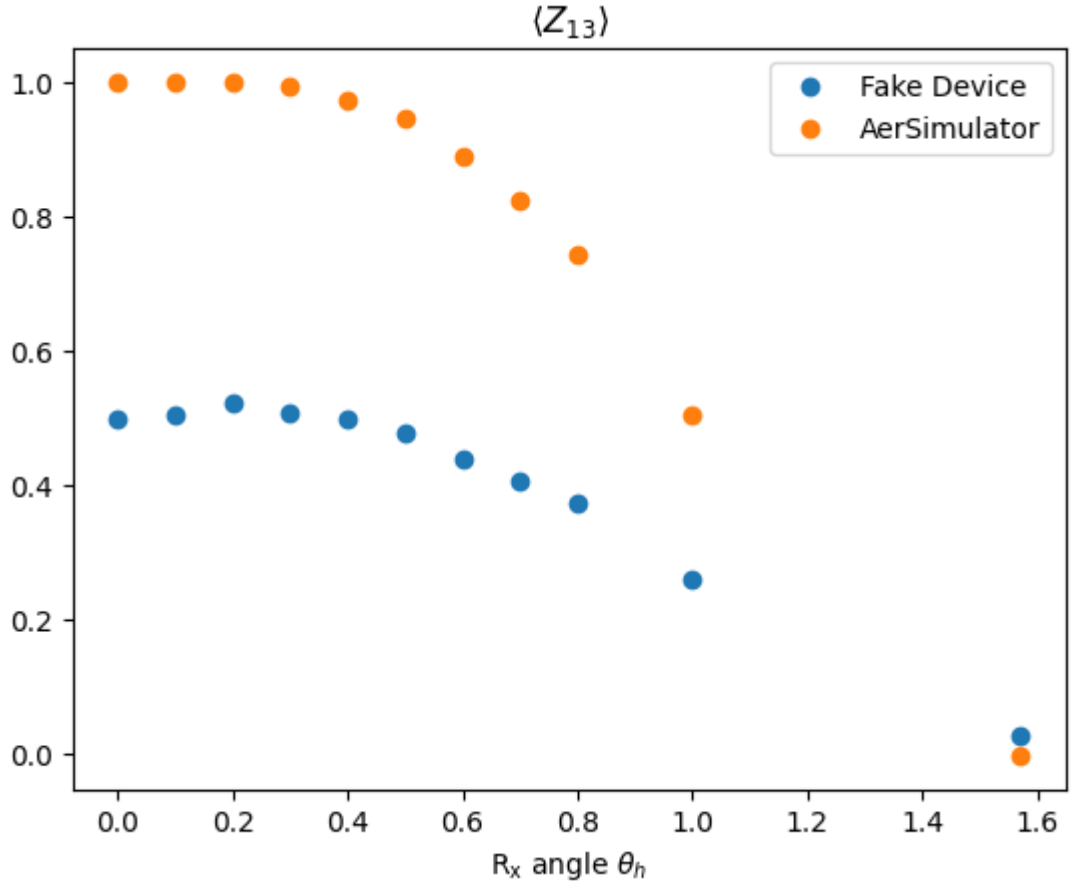
In [22]:

```
%%time
params = [[p] for p in angles]
estimator = Estimator(mode=backend_fake)
pub = (transpiled_qc_fake, transpiled_obs_fake, params)
result_fake = estimator.run([pub]).result()
```

```
CPU times: user 11min 55s, sys: 6.91 s, total: 12min 2s
Wall time: 7min 20s
```

In [23]:

```
1 plt.plot(angles, result_fake[0].data.evs, "o", label="Fake Device")
2 plt.plot(angles, result_sim[0].data.evs, "o", label="AerSimulator")
3 plt.xlabel("$\\mathrm{R_x}$ angle $\\theta_h$")
4 plt.title("$\\langle Z_{13} \\rangle$")
5 plt.legend()
6 plt.show()
```



In [24]:
```
backend_map = service.backend("ibm_torino")

num_steps = 20
qc = get_utility_circuit(backend_map, num_steps)
obs = SparsePauliOp.from_sparse_list([("Z", [62], 1)], num_qubits=backend_map.num_qubits) # Eagle
angles = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, np.pi/2]
```

In [25]:
```
backend = service.backend("ibm_torino")

transpiled_qc = transpile(qc, backend, optimization_level=1, layout_method="trivial")
transpiled_obs = obs.apply_layout(layout=transpiled_qc.layout)

print(get_circuit_info(qc, reps=1))
print(get_circuit_info(transpiled_qc))
```

```
133 qubits × 60 layers (221-depth),  Gate breakdown: SDG 6000, UNITARY 6000, CX 3000, R 2660
133 qubits × 60 layers (201-depth),  Gate breakdown: RZ 10070, SX 5320, CZ 3000
```

In [26]:
```
params = [[p] for p in angles]
estimator = Estimator(mode=backend)
pub = (transpiled_qc, transpiled_obs, params)
job = estimator.run([pub])

print(f"job id={job.job_id()}")
```

```
job id=d161uhjmk80s73a0oop0
```

In [27]:
```
# REPLACE WITH YOUR OWN JOB IDS
job = service.job("d161uhjmk80s73a0oop0")
```

In [ ]:
```
# get results
result = job.result()
print(result)
```

In [ ]:

```
result_paper = [1.0171, 1.0044, 0.9563, 0.9602, 0.8394, 0.8120, 0.5466, 0.4556, 0.1953, 0.0141, 0.0117]

# REPLACE WITH YOUR OWN JOB ID
job = service.job("d161uhjmk80s73a0oop0")

plt.plot(angles, job.result()[0].data.evs, "o", label=f"{job.backend().name}")
plt.plot(angles, result_paper, "o", label="Utility Paper")
plt.xlabel("$\\mathrm{R_x}$ angle $\\theta_h$")
plt.title("$\\langle Z_{62} \\rangle$")
plt.legend()
plt.show()
```

In [ ]:

```

```