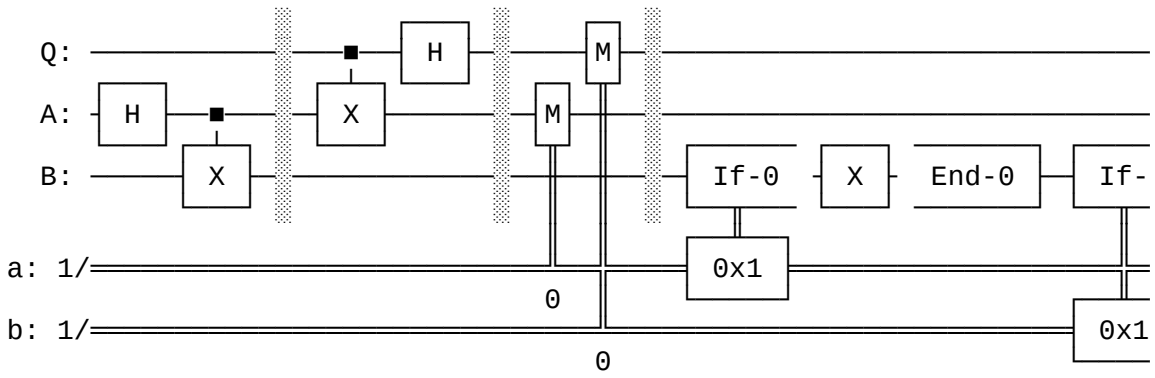
In [1]:

```
# Required imports

from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegi
from qiskit_aer import AerSimulator
from qiskit.visualization import plot_histogram
from qiskit.result import marginal_distribution
from qiskit.circuit.library import UGate
from numpy import pi, random
```

In [2]:

```
qubit = QuantumRegister(1, "Q")
ebit0 = QuantumRegister(1, "A")
ebit1 = QuantumRegister(1, "B")
a = ClassicalRegister(1, "a")
b = ClassicalRegister(1, "b")

protocol = QuantumCircuit(qubit, ebit0, ebit1, a, b)

# Prepare ebit used for teleportation
protocol.h(ebit0)
protocol.cx(ebit0, ebit1)
protocol.barrier()

# Alice's operations
protocol.cx(qubit, ebit0)
protocol.h(qubit)
protocol.barrier()

# Alice measures and sends classical bits to Bob
protocol.measure(ebit0, a)
protocol.measure(qubit, b)
protocol.barrier()

# Bob uses the classical bits to conditionally apply gates
with protocol.if_test((a, 1)):
    protocol.x(ebit1)
with protocol.if_test((b, 1)):
    protocol.z(ebit1)

display(protocol.draw())
```




```
«
«  Q: ──────────
«
«  A: ──────────
«
«  B:   End-0  ─
«
«a: 1/═══════════
«
«b: 1/═══════════
«
```

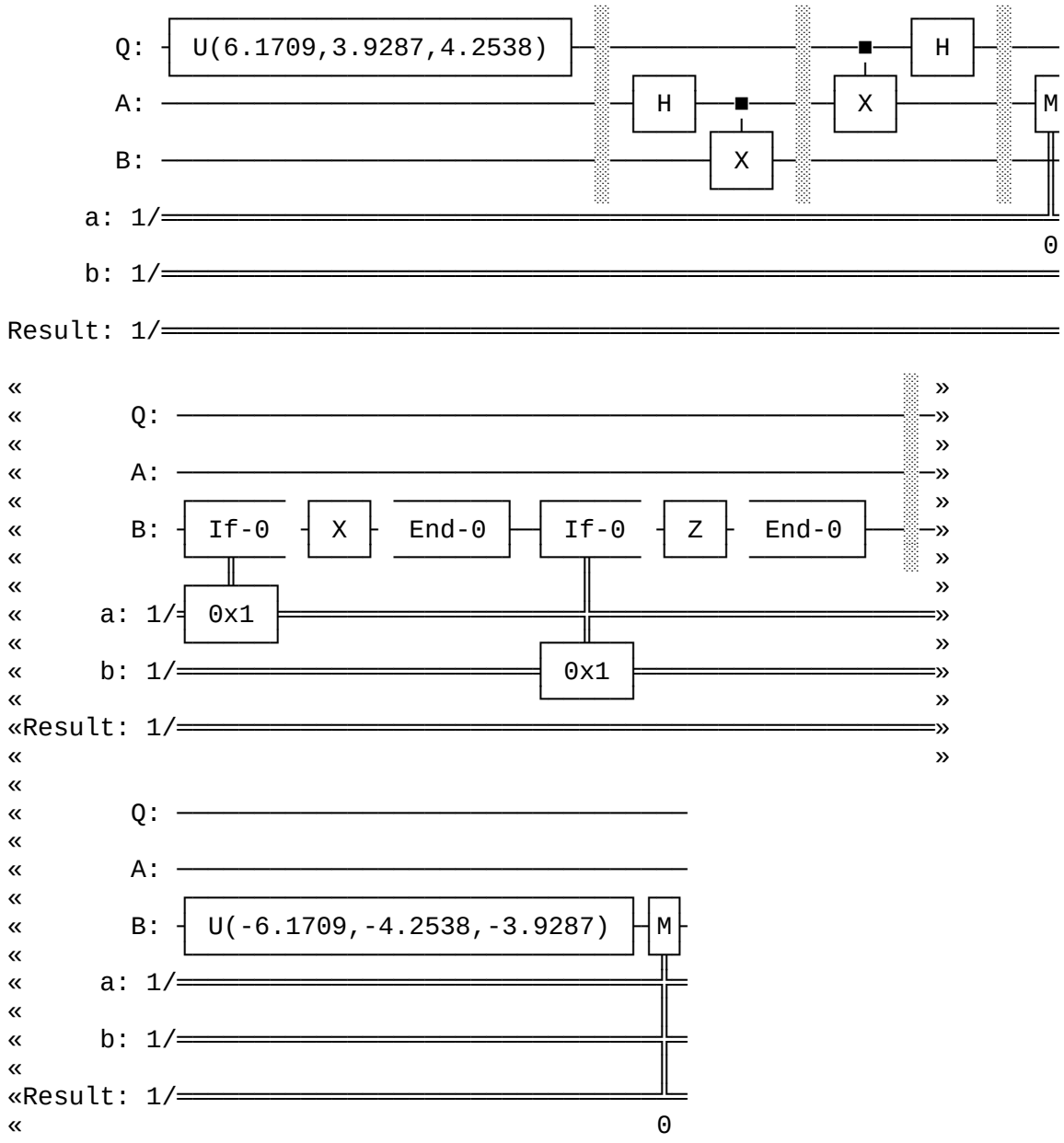
In [3]:

```
random_gate = UGate(
    theta=random.random() * 2 * pi,
    phi=random.random() * 2 * pi,
    lam=random.random() * 2 * pi,
)

display(random_gate.to_matrix())
```

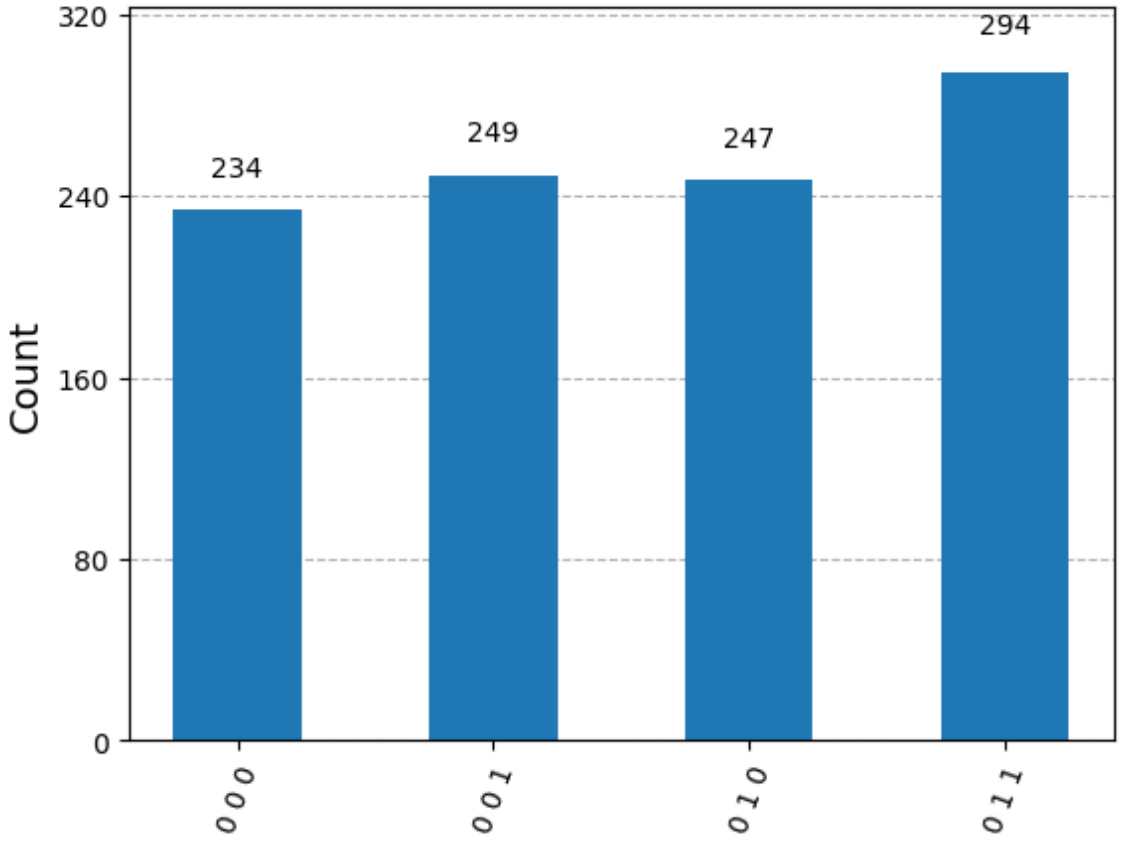
```
array([[-0.99842388+0.j        ,  0.02484455+0.05032405j],
       [-0.03961885-0.03975057j,  0.32208649-0.94504525j]])
```

In [4]:

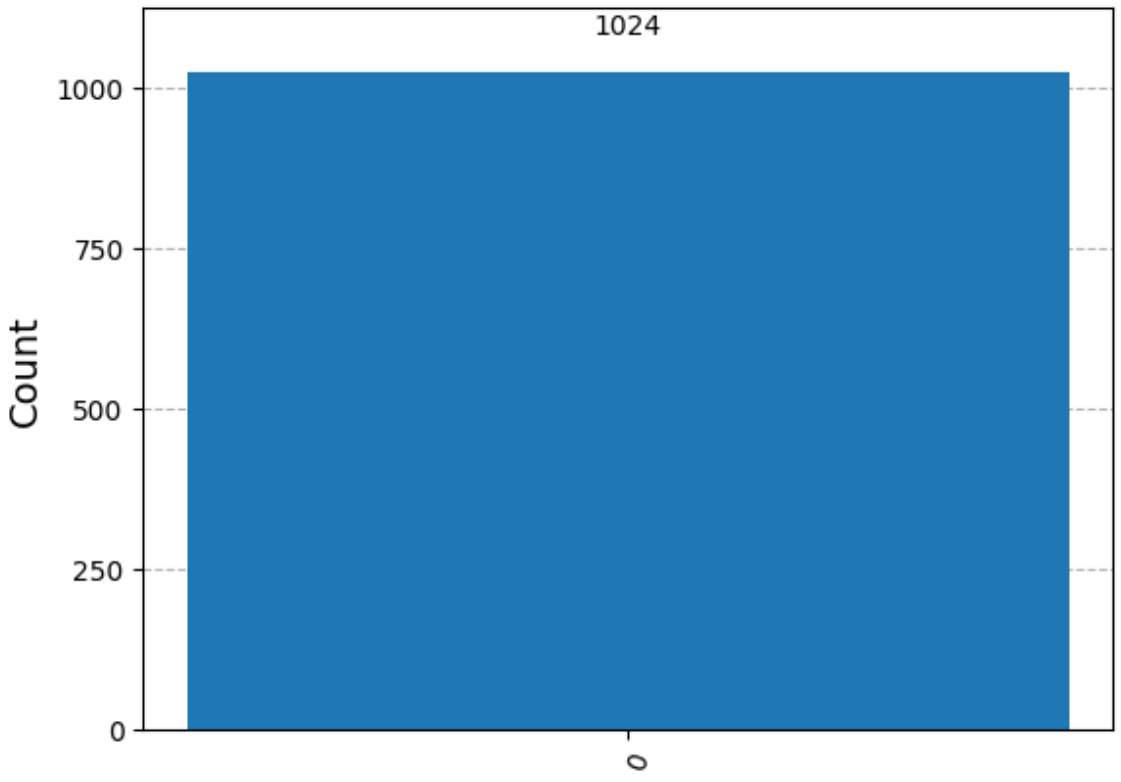
```
# Create a new circuit including the same bits and qubits used ir
# teleportation protocol.

test = QuantumCircuit(qubit, ebit0, ebit1, a, b)

# Start with the randomly selected gate on Q

test.append(random_gate, qubit)
test.barrier()

# Append the entire teleportation protocol from above.

test = test.compose(protocol)
test.barrier()

# Finally, apply the inverse of the random unitary to B and measu

test.append(random_gate.inverse(), ebit1)

result = ClassicalRegister(1, "Result")
test.add_register(result)
test.measure(ebit1, result)

display(test.draw())
```


Q:
U(6.1709,3.9287,4.2538)
H
H
A:
X
X
M
B:
a: 1/
0
b: 1/
Result: 1/
Q:
A:
B:
If-0
X
End-0
If-0
Z
End-0
a: 1/
0x1
b: 1/
0x1
Result: 1/
Q:
A:
B:
U(-6.1709,-4.2538,-3.9287)
M
a: 1/
b: 1/
Result: 1/
0

In [5]:
```
1 result = AerSimulator().run(test).result()
2 statistics = result.get_counts()
3 display(plot_histogram(statistics))
```



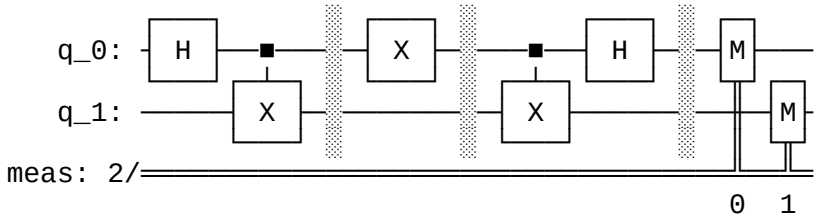
In [6]:
```
1 filtered_statistics = marginal_distribution(statistics, [2])
2 display(plot_histogram(filtered_statistics))
```



In [8]:
```
# Required imports

from qiskit import QuantumCircuit, QuantumRegister, ClassicalReg
from qiskit_aer.primitives import Sampler
from qiskit_aer import AerSimulator
from qiskit.visualization import plot_histogram
```

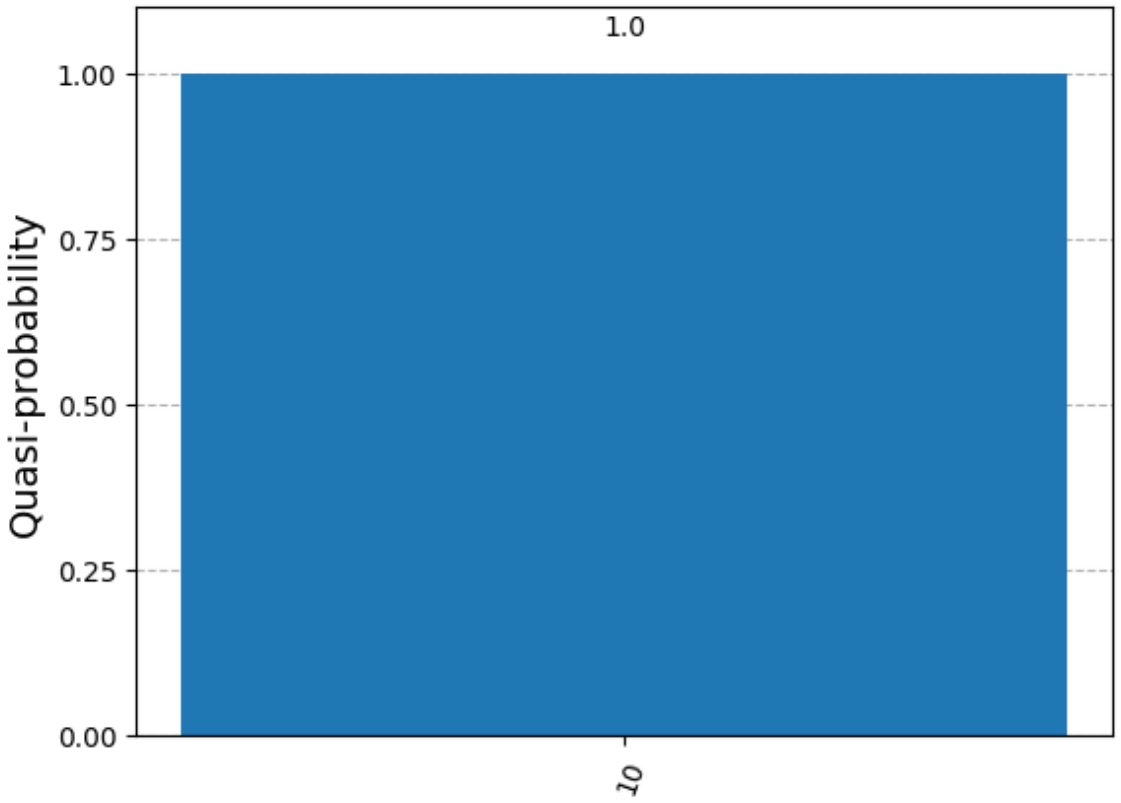
In [9]:
```
c = "1"
d = "0"
```

In [10]:
```
protocol = QuantumCircuit(2)

# Prepare ebit used for superdense coding
protocol.h(0)
protocol.cx(0, 1)
protocol.barrier()

# Alice's operations
if d == "1":
    protocol.z(0)
if c == "1":
    protocol.x(0)
protocol.barrier()

# Bob's actions
protocol.cx(0, 1)
protocol.h(0)
protocol.measure_all()

display(protocol.draw())
```



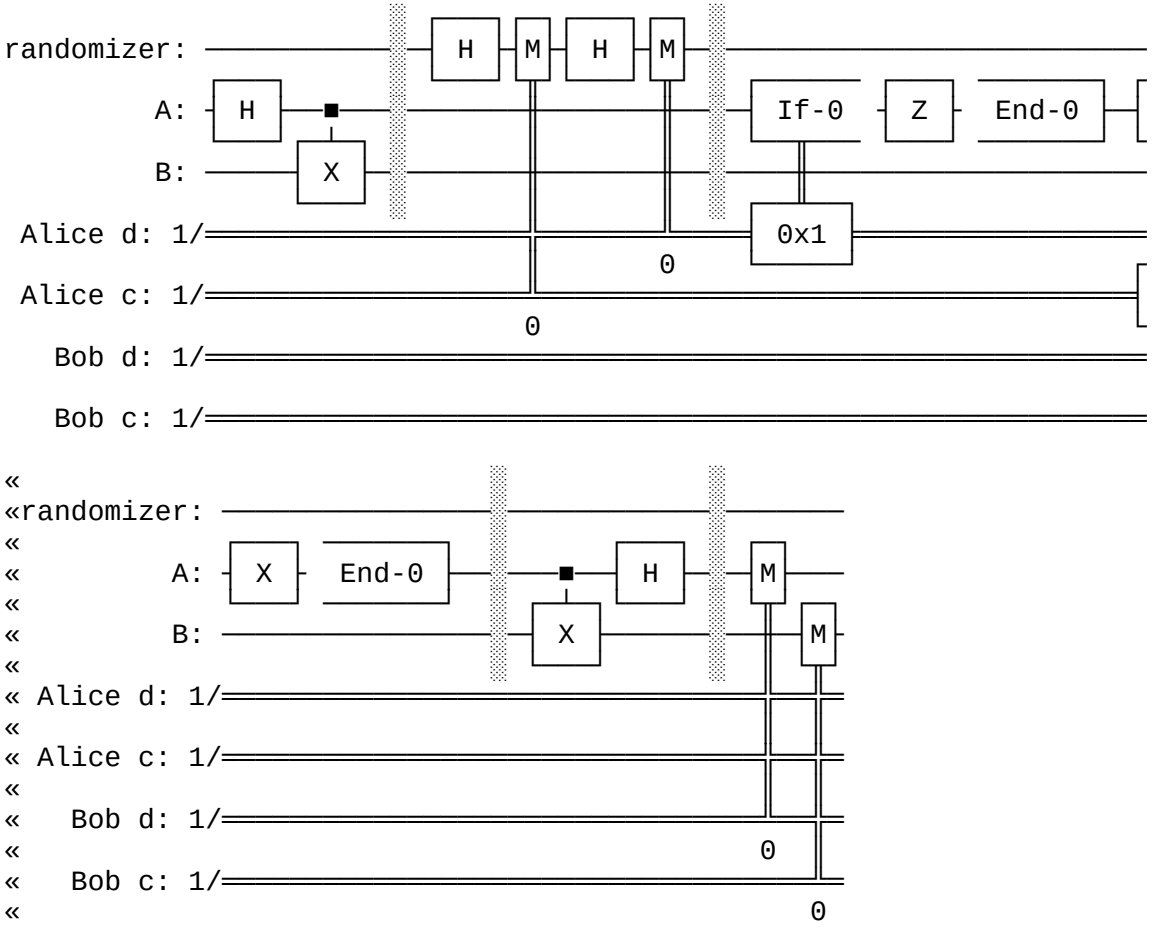
In [11]:
```
result = Sampler().run(protocol).result()
statistics = result.quasi_dists[0].binary_probabilities()

for outcome, frequency in statistics.items():
    print(f"Measured {outcome} with frequency {frequency}")

display(plot_histogram(statistics))
```

```
Measured 10 with frequency 1.0
```

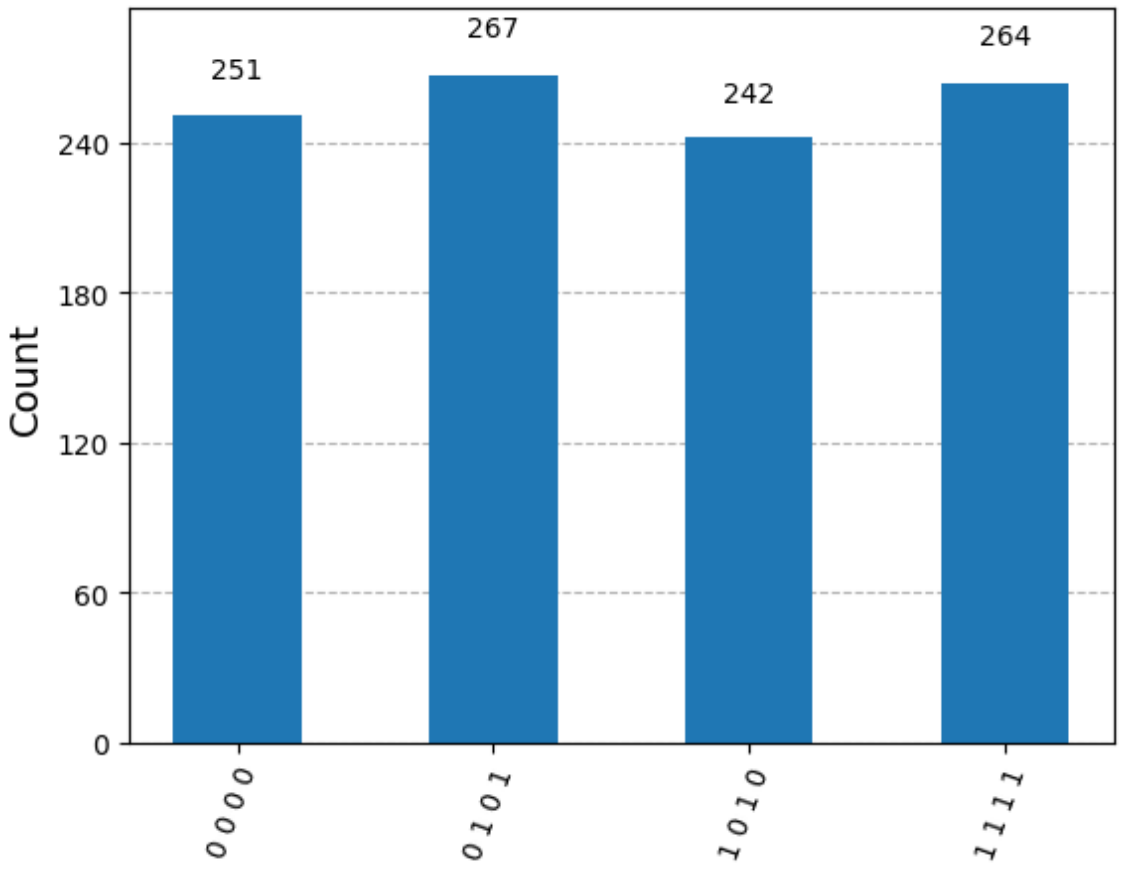


In [12]:

```
rbg = QuantumRegister(1, "randomizer")
ebit0 = QuantumRegister(1, "A")
ebit1 = QuantumRegister(1, "B")

Alice_c = ClassicalRegister(1, "Alice c")
Alice_d = ClassicalRegister(1, "Alice d")

test = QuantumCircuit(rbg, ebit0, ebit1, Alice_d, Alice_c)

# Initialize the ebit
test.h(ebit0)
test.cx(ebit0, ebit1)
test.barrier()

# Use the 'randomizer' qubit twice to generate Alice's bits c and
test.h(rbg)
test.measure(rbg, Alice_c)
test.h(rbg)
test.measure(rbg, Alice_d)
test.barrier()

# Now the protocol runs, starting with Alice's actions, which dep
# on her bits.
with test.if_test((Alice_d, 1), label="Z"):
    test.z(ebit0)
with test.if_test((Alice_c, 1), label="X"):
    test.x(ebit0)
test.barrier()

# Bob's actions
test.cx(ebit0, ebit1)
test.h(ebit0)
test.barrier()

Bob_c = ClassicalRegister(1, "Bob c")
Bob_d = ClassicalRegister(1, "Bob d")
test.add_register(Bob_d)
test.add_register(Bob_c)
test.measure(ebit0, Bob_d)
test.measure(ebit1, Bob_c)

display(test.draw())
```



In [13]:
```
1 result = AerSimulator().run(test).result()
2 statistics = result.get_counts()
3 display(plot_histogram(statistics))
```



In [14]:
```
# Required imports

from qiskit import QuantumCircuit
from qiskit_aer.primitives import Sampler
from numpy import pi
from numpy.random import randint
```

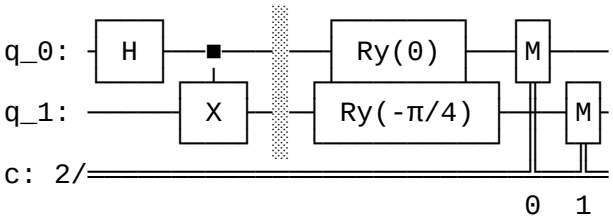
In [15]:
```
def chsh_game(strategy):
    """Plays the CHSH game
    Args:
        strategy (callable): A function that takes two bits (as `
            returns two bits (also as `int`s). The strategy must
            rules of the CHSH game.
    Returns:
        int: 1 for a win, 0 for a loss.
    """
    # Referee chooses x and y randomly
    x, y = randint(0, 2), randint(0, 2)

    # Use strategy to choose a and b
    a, b = strategy(x, y)

    # Referee decides if Alice and Bob win or lose
    if (a != b) == (x & y):
        return 1  # Win
    return 0  # Lose
```

In [16]:

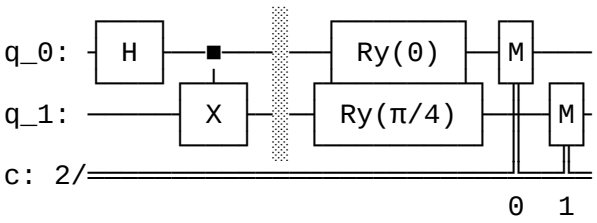
```
def chsh_circuit(x, y):
    """Creates a `QuantumCircuit` that implements the best CHSH s
    Args:
        x (int): Alice's bit (must be 0 or 1)
        y (int): Bob's bit (must be 0 or 1)
    Returns:
        QuantumCircuit: Circuit that, when run, returns Alice and
            answer bits.
    """
    qc = QuantumCircuit(2, 2)
    qc.h(0)
    qc.cx(0, 1)
    qc.barrier()

    # Alice
    if x == 0:
        qc.ry(0, 0)
    else:
        qc.ry(-pi / 2, 0)
    qc.measure(0, 0)

    # Bob
    if y == 0:
        qc.ry(-pi / 4, 1)
    else:
        qc.ry(pi / 4, 1)
    qc.measure(1, 1)

    return qc
```

In [17]:
```
# Draw the four possible circuits

print("(x,y) = (0,0)")
display(chsh_circuit(0, 0).draw())

print("(x,y) = (0,1)")
display(chsh_circuit(0, 1).draw())

print("(x,y) = (1,0)")
display(chsh_circuit(1, 0).draw())

print("(x,y) = (1,1)")
display(chsh_circuit(1, 1).draw())
```

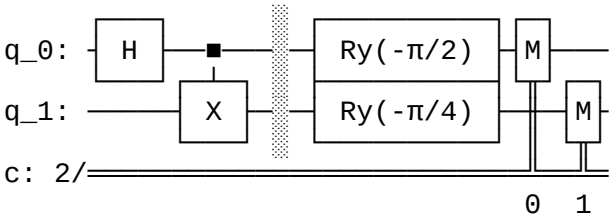
```
(x,y) = (0,0)
```



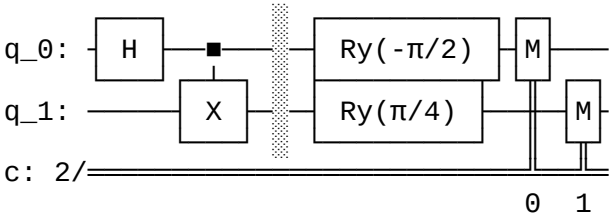

```
(x,y) = (0,1)
```




```
(x,y) = (1,0)
```




```
(x,y) = (1,1)
```



In [18]:
```
sampler = Sampler()


def quantum_strategy(x, y):
    """Carry out the best strategy for the CHSH game.
    Args:
        x (int): Alice's bit (must be 0 or 1)
        y (int): Bob's bit (must be 0 or 1)
    Returns:
        (int, int): Alice and Bob's answer bits (respectively)
    """
    # `shots=1` runs the circuit once
    result = sampler.run(chsh_circuit(x, y), shots=1).result()
    statistics = result.quasi_dists[0].binary_probabilities()
    bits = list(statistics.keys())[0]
    a, b = bits[0], bits[1]
    return a, b
```

In [19]:
```
NUM_GAMES = 1000
TOTAL_SCORE = 0

for _ in range(NUM_GAMES):
    TOTAL_SCORE += chsh_game(quantum_strategy)

print("Fraction of games won:", TOTAL_SCORE / NUM_GAMES)
```

```
Fraction of games won: 0.837
```

In [20]:
```
def classical_strategy(x, y):
    """An optimal classical strategy for the CHSH game
    Args:
        x (int): Alice's bit (must be 0 or 1)
        y (int): Bob's bit (must be 0 or 1)
    Returns:
        (int, int): Alice and Bob's answer bits (respectively)
    """
    # Alice's answer
    if x == 0:
        a = 0
    elif x == 1:
        a = 1

    # Bob's answer
    if y == 0:
        b = 1
    elif y == 1:
        b = 0

    return a, b
```

In [21]:
```
NUM_GAMES = 1000
TOTAL_SCORE = 0

for _ in range(NUM_GAMES):
    TOTAL_SCORE += chsh_game(classical_strategy)

print("Fraction of games won:", TOTAL_SCORE / NUM_GAMES)
```

```
Fraction of games won: 0.765
```

In [ ]:
```

```